EFFECTS OF TEMPORAL VARIATIONS

IN LEAF ANGLE ON

BACKSCATTER AND INVERSION

BY

R. H. Lang
N. S. Chauhan

Department of Electrical Engineering
School of Engineering and Applied Science
The George Washington University
Washington D.C.
June 1988

prepared for:

National Aeronautics and Space Administration
Headquarters,
Land Processor Branch, Code EE
Washington, D.C. 20546
NAGW-984

Abstract

The effects of time-dependent changes in the canopy geometry on the backscattering have been studied through microwave modeling. Soybeans leaves have been monitored round the clock for any changes in their inclination angle. The angle distribution densities for morning and afternoon periods are inputted to the microwave model based on the distorted Born approximation and the backscattering coefficients for horizontal, vertical and cross-polarization are obtained. Sensitivity of backscattering to leaf angle distribution is discussed in the context of the non-simultaneity of observations.

Linear inversion of leaf angle is carried out using Phillips-Twomey technique under the assumption that skin depth of the vegetation is large. Even for a 50 percent noise, the leaf angle density distribution for morning and evening period is clearly distinguishable. All these single polarization inversions require data at approximately 10 incident angles. Since radar observations from space are usually limited to 3 or 4 incident angles, a method combining the three polarization data has been devised which reduces the number of incident angles required by a factor of three.

TABLE OF CONTENTS

Section 1 - Introduction.......................................1

Section 2 - Formulation of the Direct Problem........................2

Section 3 - Data Analysis.......................................4

Section 4 - Results and Discussion.................................5

Section 5 - Inverse Problem.......................................6

Section 6 - Results and Discussion of Inversion.......................9

Section 7 - References...12

Figures...13

Appendix A - Soybean Leaf Data

Appendix B - Polarization Utilization of the Microwave Inversion of Leaf Angle Distribution

1. Introduction

In the global ecosystem, the vegetation plays an important part in the determination of water and energy estimates. Next to the ocean, vegetation is the most dynamic surface that covers the earth. The response of the vegetation to sensors have been found to be dependent upon incident angle, local meteorological condition, canopy geometry etc. An algorithm is needed to interpret the remotely sensed data under different physical and environmental conditions. This will be particularly helpful in analyzing and inverting the backscattered data from the non-simultaneous observations of the vegetation.

In the present report, the microwave response of the vegetation under non-simultaneous observations is modeled. In the past both continuous (Tsang and Kong; 1981, Jin and Kong, 1984) and discrete (Lang, 1981) random media techniques have been used for modeling. However discrete methods offer a distinct advantage as it relates the radar backscattering signal directly to the scatter characteristics such as size, orientation and water content. Following the approach outlined by Lang and Sidhu (1983), LeVine et. al (1983), the soybean canopy has been modeled in the microwave frequency regime by replacing their leaves with circular, lossy, dielectric disks. The distorted Born approximation is used to determine the backscattering coefficients of the vegetation layer in terms of the scattering amplitude of the individual scatterers. The observed inclination angle of the leaves in the form of probability density function is inputted into the model to get the scattering coefficients. The canopy modeling has been limited only to the morning and the afternoon periods just to demonstrate the effects of non-simultaneity of observations on the backscattering coefficients.

The scattering data from the model is also inverted to get back the probabiltiy distribution of the leaves for the morning and afternoon periods. Assuming a large skin depth of the vegetation reduces the distorted Born equations to a linear relationship between the backscattering coefficient and the inclination angle distribution. Due to the ill-posed nature of this relation, Phillips-Twomey regularization method with a second-order smoothing condition is used for inversion. If the data is available at too few incident angles, a multipolarization inversion algorithm is suggested to get an accurate estimate of the inclination angle of the leaves.

2. Formulation of the Direct Problem:

A layer of vegetation of thickness d is modeled by a slab of lossy dielectric disks having random positions and orientation statistics. The wave is incident at an angle θ_o and the polarization vector of the incoming and scattered wave are shown is Fig. 1. The disks of radius a and thickness h are assumed to be independent of one another. The ground under vegetation is taken to be homogeneous lossy dielectric half space and the interface between the vegetation and the ground is assumed to be flat. The geometry for one particular disk contained in the slab with its inclination angle θ and azimuthal angle ϕ is shown in Fig. 2.

The direct problem consists of computing the backscattering coefficients for a canopy of discrete scatterers by the distorted Born approximation. This can be expressed through direct σ°_{pqd}, reflected σ°_{pqr}, direct-reflected σ°_{pqdr} scattering coefficients. For like (p=q) and unlike (p≠q) polarization the backscattering coefficient is given by (Lang and Sidhu, 1983)

$$\sigma^{\circ}_{pq} = \sigma^{\circ}_{pqd} + \sigma^{\circ}_{pqr} + \sigma^{\circ}_{pqdr} \qquad (1)$$

$$p, q \in \{h, v\}$$

where σ°_{pq} is the total backscattering coefficient for a canopy of discrete scatterers, and

$$\sigma^{\circ}_{pqd} = \rho d \ \sigma_{pqd} \left[\frac{1 - e^{-2\tau}}{2\tau} \right] \qquad (2)$$

$$\sigma^{\circ}_{pqr} = \rho d \ \sigma_{pqr} \ \Gamma^{2}_{pqg} \left[\frac{e^{-2\tau} - e^{-4\tau}}{2\tau} \right] \qquad (3)$$

$$\sigma^{\circ}_{pqdr} = 4 \ \rho d \ \sigma_{pqdr} \ \Gamma_{pqg} \ e^{-2\tau} \qquad (4)$$

Here ρ is the density of the scatterers, Γ^{2}_{pqg} is the reflectivity of the ground and σ_{pqd}, σ_{pqr}, σ_{pqdr} are the average direct, reflected and direct-reflected scattering cross-sections respectively. The optical depth τ for a wave incident on the scatterer is

$$\tau = d \rho \ \sigma_{pqt} \sec \theta_{\circ} = 2 \ \lambda \ \rho \ \mathrm{Im} \ \bar{f}_{pq} \sec \theta_{\circ} \qquad (5)$$

where $\theta_{\circ}$ is the incident angle, σ_{pqt} is the extinction coefficient of a particular scatterer, λ is the wavelength, $\bar{f}_{pq}$ is the average forward scattering amplitude of the disk. The averaging is done over the disk orientation angles and the disk radius. Since the disks are assumed to be uniformly distributed in azimuthal coordinate, the average can be written as

$$\bar{f}_{pq} = \int \bar{f}_{pq}(a,\theta)\ p(a,\theta)\ d\theta\ da \qquad (6)$$

Here $p(a,\theta)$ is the joint probability density function of disk having radius a and inclination angle θ. Assuming a and θ to be independent random variable

$$p(a,\theta) = p_1(a) \quad p_2(\theta) \qquad (7)$$

Thus a change in distribution of the inclination angle will be reflected in the backscattering coefficient of the canopy. In the present study, the inclination angle distribution of the soybeans as measured from field experiment is inputted into microwave model and its effects on the backscattering coefficients are analyzed in the context of the simultaneity experiment.

3. Data Analysis

The soybean leaf data was collected at the University of Michigan by Criag Dobson of University of Michigan and Jobea Cimino of Jet Propulsion Laboratory in the summer (August 18-24) of 1987 (Appendix A). The soybeans which had been planted carefully and watered periodically were mature during this part of the year. In the canopy geometry of the plant there are three leaflets attached to each stem (petiole) of the soybean plant. In the present experiment six stems each containing three leaflets were chosen. The stems or the leaves chosen do not represent any particular position on the plants, rather these are uniformly distributed over the whole the length and breadth of the plant. For the marked leaves the inclination angles were measured round the clock at 1-2 hours intervals for 4 days. For the purpose of analysis, the data has been grouped into morning and afternoon periods. The morning data consists of data taken at 0613, 0640, 0703, 0734 Hrs. local time and afternoon data is the data taken at 1418, 1438, 1445, 1455 Hrs. local time. Fig. 3 and Fig. 4 show the histogram of the leaves for the morning and the afternoon periods respectively. A clear distinction in the inclination angle is seen for the morning and afternoon periods.

4. Results and Discussion

The model parameters are chosen to represent a mature soybean crop similar to the one for which data was taken. The dielectric constant ϵ_r = 39.1 + i8.28 at frequency of 4GHz is chosen following the deLoor's model. The disks have radius of 4cm and thickness 0.2mm. The density of the disks is $1000 m^{-3}$ and the disks are assumed to be placed in a slab of 0.6m thick. The probability density for the inclination angle has been approximated as

$$p_2(\theta) = 0.7 \cos \theta + 0.3 \sin 2 \theta \tag{8}$$

for morning

and

$$p_2(\theta) = 0.9 \sin \theta + 0.1 \sin 2 \theta \tag{9}$$

for afternoon

These two probability densities have a trend which is close to those observed and shown in the histograms (Fig. 3,4).

The backscattering modeling results for the horizontal, vertical and cross-polarization are shown in Figs. 5-7 for both the morning and the afternoon periods. The probability distribution of the leaves given by Eqs. (8) and (9) suggest that in the morning the leaves are mostly horizontal while during the afternoon period the leaves become vertical due to the heating from the sun. Both for the horizontal and the vertical polarizations, the largest changes in backscattering coefficients occur at large angles of incidence, while for cross-polarization the morning-afternoon differences are large at low-angles. Thus at large angles, the horizontal leaves for the horizontal and the vertical polarization give stronger backscatter returns and makes themselves distinguishable from the vertical leaves. It has been found that distinguishability of the leaves is also dependent upon frequency used. At lower frequency (=1.5 GHz), the morning-afternoon differences in the backscattering coefficient reduces from an average value of 2.5DB to 1.5DB (Figs. 8-10) It has been assumed here that as the day progresses, the dielectric constant both for the leaves and soil do not change, though, in the strict sense none of these remain constant with time (Dobson and Ulaby, 1986). In fact in some seasons, there can be deposition of dew on the leaves that will change the scatter characteristics of the canopy.

5. Inverse Problem

Equations (1)-(4) suggest a non-linear relationship between the backscattering coefficient and the joint probability density distribution. If the skin depth ($\frac{1}{\tau}$) for the agricultural crops is large i.e. $\tau << 1$, then the distorted Born approximation, Eqs.(2)-(4), reduced to the following Born equations:

$$\sigma^{\circ}_{pqd} = \rho \, d \, \sigma_{pqd} \tag{10}$$

$$\sigma^{\circ}_{pqdr} = \rho d \, \sigma_{pqdr} \, \Gamma^{2}_{pqg} \tag{11}$$

$$\sigma^{\circ}_{pqdr} = 4\rho d \, \sigma_{pqdr} \, \Gamma_{pqg} \tag{12}$$

An examination of Eqs.(10)-(12) shows that $p(a,\theta)$ is now linearly related to the backscattering coefficient. In the present frequency range, the skin depth of vegetation is large at low angles of incidence, thus, the assumption

$\tau << 1$ gives a linear relationship between probability density and backscattering coefficient. Non-linear inversion of leaf angle distribution is in progress. However, in this report, the results of only linear inversion will be given. Symbolically one can write the relationship between backscattering coefficient and probability density distribution as:

$$\sigma^{\circ}_{pq}(\theta_{\circ}) = \int K_{pq}(\theta_{\circ}, a, \theta)\, p(a,\theta)\, da\, d\theta \tag{13}$$

Here K_{pq} is the kernel function and contains all the three direct, reflected and direct reflected parts:

$$K_{pq} = K_{pqd} + K_{pqr} + K_{pqdr} \tag{14}$$

Explicit expression for different kernels are given in Appendix B.

The integral equation (13) relates the backscattering coefficient to the joint distribution over the disk radius and the inclination angle. Since the radius distribution will be assumed as known a priori, Eq.(13) can be rewritten as

$$\sigma^{\circ}_{pq}(\theta_{\circ}) = \int K^{(\theta)}_{pq}(\theta_{\circ},\theta)\, p(\theta)\, d\theta \tag{15}$$

where

$$K^{(\theta)}_{pq}(\theta_{\circ},\theta) = \int K_{pq}(\theta_{\circ},a,\theta)\, p(a|\theta)\, da \tag{16}$$

Here $p(a|\theta)$ is the conditional probability of radius given that inclination angle is known.

The inverse problem involves the determination of the probability density function for the disk inclination $p(\theta)$, given the backscattering coefficients at different angles of incidence. To invert Eq.(15), the continuous variables are discretized (See Appendix B), and Eq.(15) is approximated by a linear system of equations

$$\sigma = B\ p \tag{17}$$

where

$$\sigma = \left[\sigma^{\circ}_{pq}(\theta_{i1}), \ldots\ldots\ldots\ldots \sigma_{pq}(\theta_{iM})\right]^T \tag{18}$$

$$p = \left[p(\theta_1), \ldots\ldots\ldots\ldots p(\theta_N)\right]^T \tag{19}$$

$$B = \left[B_{mn}\right], \qquad B_{mn} = K(\theta_{im}, \theta_n) \tag{20}$$

$$m = 1,2\ldots.M, \qquad n = 1,2,\ldots.N. \tag{21}$$

Here superscript T stands for transpose.

The integral equation (15) is ill-posed i.e. the small changes is σ gives large changes in p. Therefore, a mapping of σ to p must be regularized in the inverse problem. The Phillips-Twomey technique with second-order smoothing condition is used to get the regularized solution. This is given by

$$p_\alpha = (B^T B + \alpha H)^{-1} B^T \sigma_\delta \tag{22}$$

where H is a matrix arising from second order smoothing condition, α is the regularization parameter and σ_δ represents the inaccurate data. δ is a measure of noise in the data. Eq.(22) is solved for a monotonic sequence of α values and the best α is chosen such that the smoothest solution does not exceed the noise limit i.e. $||\sigma_\alpha - \sigma_\delta|| < \delta$. σ_α represents the backscattering coefficient for a given value of regularization parameter.

The inversion has been carried out for morning and afternoon leaf angle distribution for the horizontal, vertical and cross-polarized cases. It has also been shown that if the data is available at too few incident angles, the inverted probability density gives a poor estimate of the actual density. The estimation can be greatly improved by inverting the combined three polarization data. In this case, the kernel used in Eq.(13) will be set for all the three combined polarizations rather than for single polarization (See Appendix B). If the backscattering coefficients for a particular polarization type are measured at M incident angle and Eq. (17) is evaluated at each of these angles then a system of M linear equations with N unknowns is generated. The kernel in this case is MxN matrix. For the backscattering coefficients sampled at L incident angles, a system of M = 3L equations can be generated. In this case only one third the number of incident angles will be required compared to using one polarization type alone.

6. Results and Discussion of Inversion

The disks have been assumed to be of only one size, so the probability distribution of the radii is written as $p(a) = \delta(a-a_o)$, where a_o represent the average radius = 4 cm of the disk. The backscattering coefficients have been corrupted by 5 percent noise to simulate the observed measured data.

Figs. 11-16 shows the behavior of the backscattering coefficient as determined found by Born approximation Eqs. (10)-(12) and its discretized form Eq. (17) as a function of incidence angle. The backscatter data is assumed to ~~be~~ available at nine angles of incidences and is discretized, after every ten degree in the range of 5-85 degrees. For all the three polarizations, the discretized and the continuous values during the morning and afternoon period are close to one another (Figs. 11-16).

Corrupting the discretized backscattering coefficient with 5 percent noise, the inversion is carried out to determine the probability density for a sequence of regularization parameters. Figs. 17-19 show the actual and the inverted inclination angle probability density for the morning period for the best value of the regularization parameter α. It is seen that for all the three polarizations, the inverted probability density shows a trend quite similar to the acutal probability density. The agreement is better for the like polarization, compared to the cross-polarization. The whole procedure is repeated for the afternoon distribution of the leaf inclination angle. It is noticed from Figs. 20-22 that as is in case of morning inclination angle distribution of the soybean leaves, the evening inclination angle distribution can also be determined through the present inversion technique.

The robustness of the Phillips-Twomey inversion technique has been tested with a noise of 50 percent added to the backscattering coefficient for the horizontal polarization. From the trend of the inverted inclination angle distributions (Figs. 23-24), one can still notice the effect of non-simultaneity in the radar observations on the soybeans.

For most of the space-born remote sensing observations, the number of incident angles is not large (Cimino, 1986). For example, in SIR-B experiment, the number of angles was just four. For such a practical applications, it becomes imperative to look at the inverted results with a few incident angles. To study this, the backscattered data is discretized at three angles i.e. 15°, 45°, 75°. The regularized solution (Eq. (22)) is obtained for a 5 percent noise for the three incident angles. It is seen that the three point inversion (Fig. 25-26) gives a poor estimate of the inclination angle probability for any one polarization. To get a better estimate, a technique which involves the utilization of the data sampled at three polarization and at three incident angles, is attempted (See Appendix B for details). The kernal for the combined polarization is set up for the nine inclination angles and the three incident angles (for each polarization). The backscattering coefficients for the combined polarization corrupted with 5 percent noise is inverted to get the probability density. A much better agreement between the true and inverted probability density is reached in this case (Fig. 27). Quantitatively, the combined polarization inversion for the three point case is about 10 times more accurate than the three point single polarization inversion (See Appendix B). Fig. 28 shows the three-point inverted probability density for the afternoon period.

The present report showed that relationship between non-simultaneity of observations from the vegetation. The study can be easily extended to see the day and night relationship between the leaves inclination angles and its corresponding backscattering coefficient. The effect of local meteorlogical condition can also be similarly studied to improve the present day understanding of the response of the vegetation to the microwaves.

7. References

Cimino J.R., 1986, The SIR-B Multiple-Incidence Angle Experiment over Argentina and Chile, The Second Space-Born Imaging Radar Symposium, Jet Propulsion Laboratory, Pasadena, CA.

Dobson M.C. & Ulaby F.T.,1986, Active Microwave Soil Moisture Research, IEEE Trans. Geosci & Remote Sensing, 24, 23-36.

Jin Y. & Kong J, 1984, Strong Fluctuation Theory for Electromagnetic Wave Scattering by a Layer of Random Discrete Scatterers, J. Appl. Phys.,55, 1364-1369.

Lang R.H., 1981, Electromagnetic Backscattering From a Sparse Distribution of Lossy Dielectric Scatterers, Radio Sci, 16, 15-30.

Lang R.H. & Sidhu J.S., 1983, Electromagnetic Backscattering From a Layer Vegitation: A Discrete Approach, IEEE Trans. Geosci & Remote Sensing, 21, 62-71.

Lang R.H. & Saleh H.A., 1985, Microwave Inversion of Leaf Area and Inclination Angle Distribution form Backscattered Data, IEEE Trans Geosci & Remote Sensing, 23, 685-694.

LeVine D.M., Meneghini R, Lang R.H.& Seker S.S, 1983, Scattering from Arbitrarily Oriented Dielectric Disks in the Physical Optics Regime, J. Opt. Soc. Am., 73, 1255-62.

Tsang L.& Kong J, 1981, Application of Strong Fluctuation Random Media Theory to Scattering from Vegitation like Half-Space, IEEE-GE, 19, 63-69.

Ulaby F.T., 1977, Radar Measurement in Agricultural and Forestry Application, RSL-Technical Report 330-5, University of Kansas Center for Research, Inc.

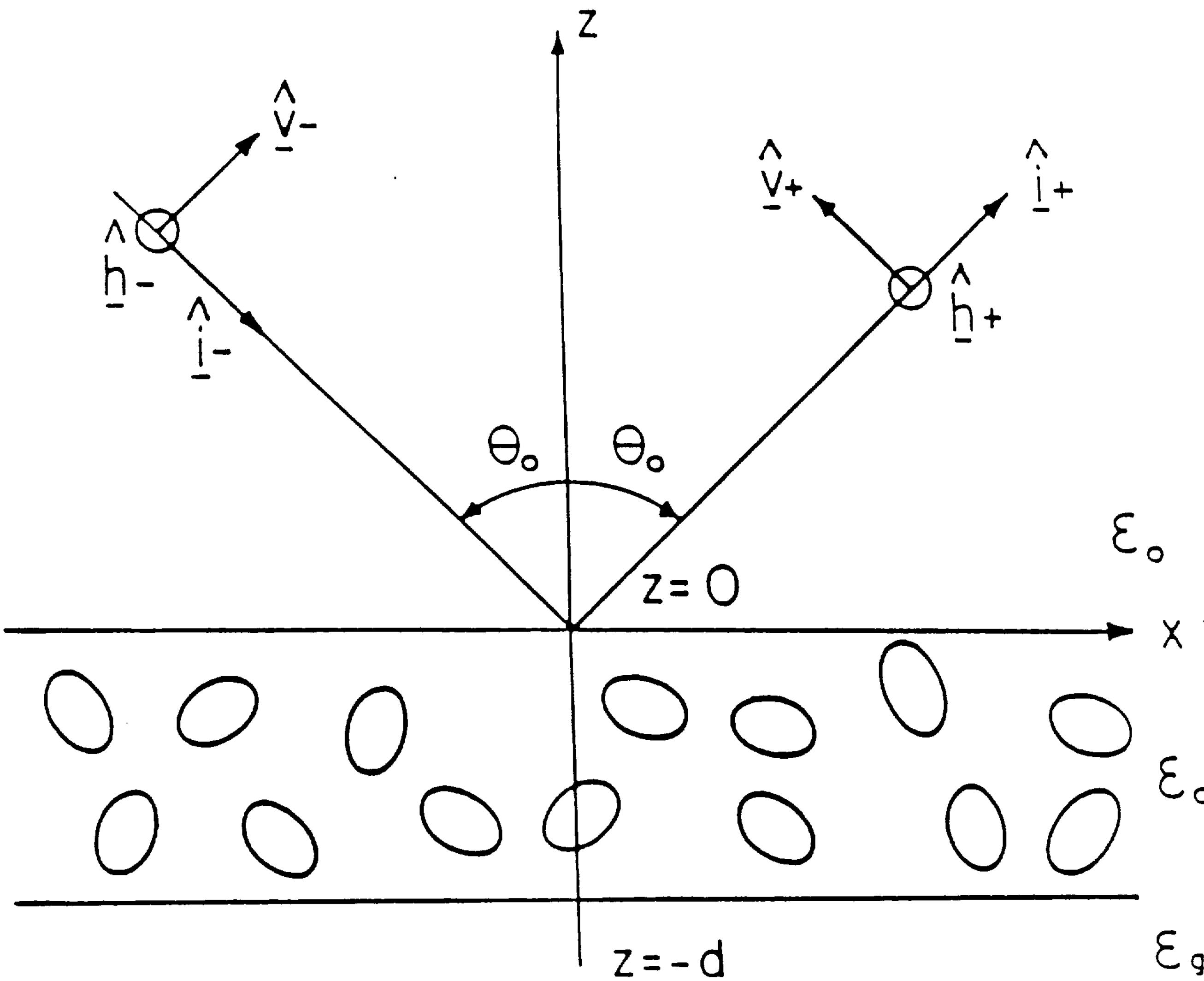


Fig. 1 Soybean Canopy Model.

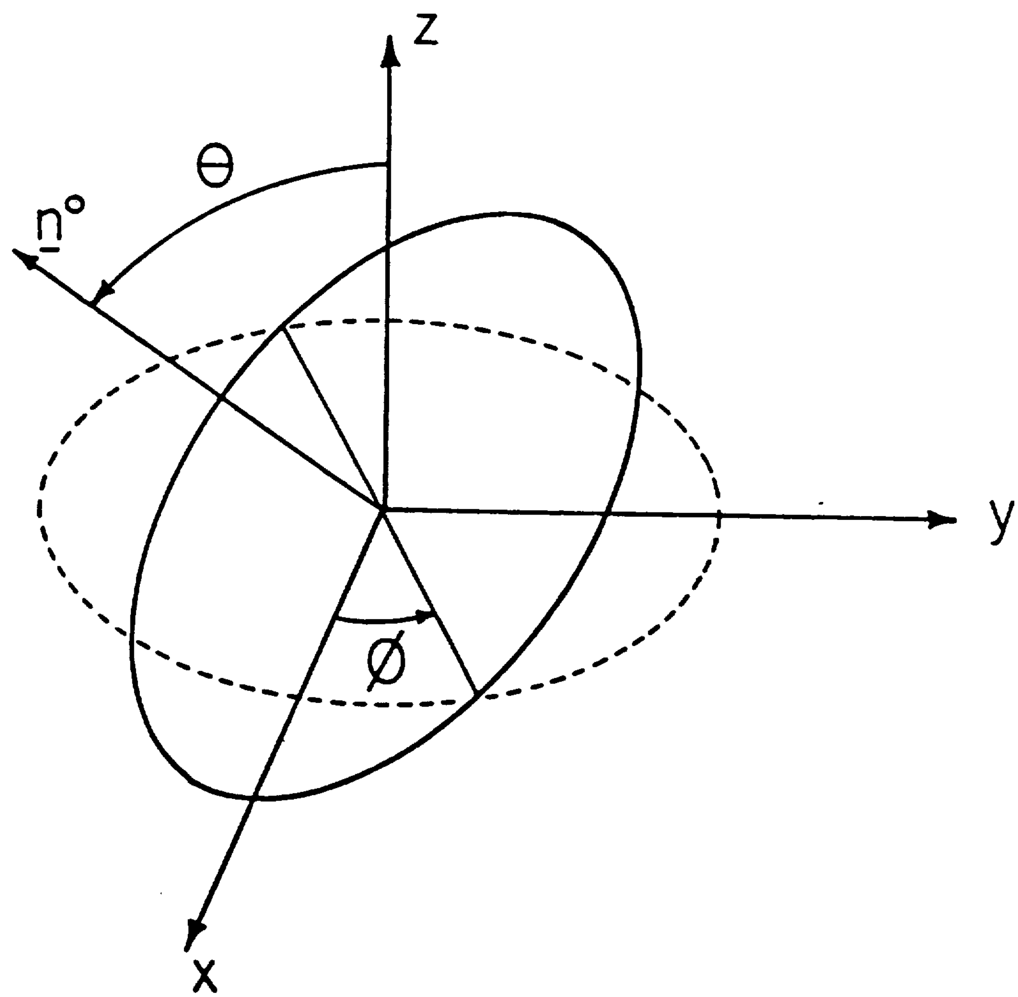


Fig. 2 Single Scatter Description.

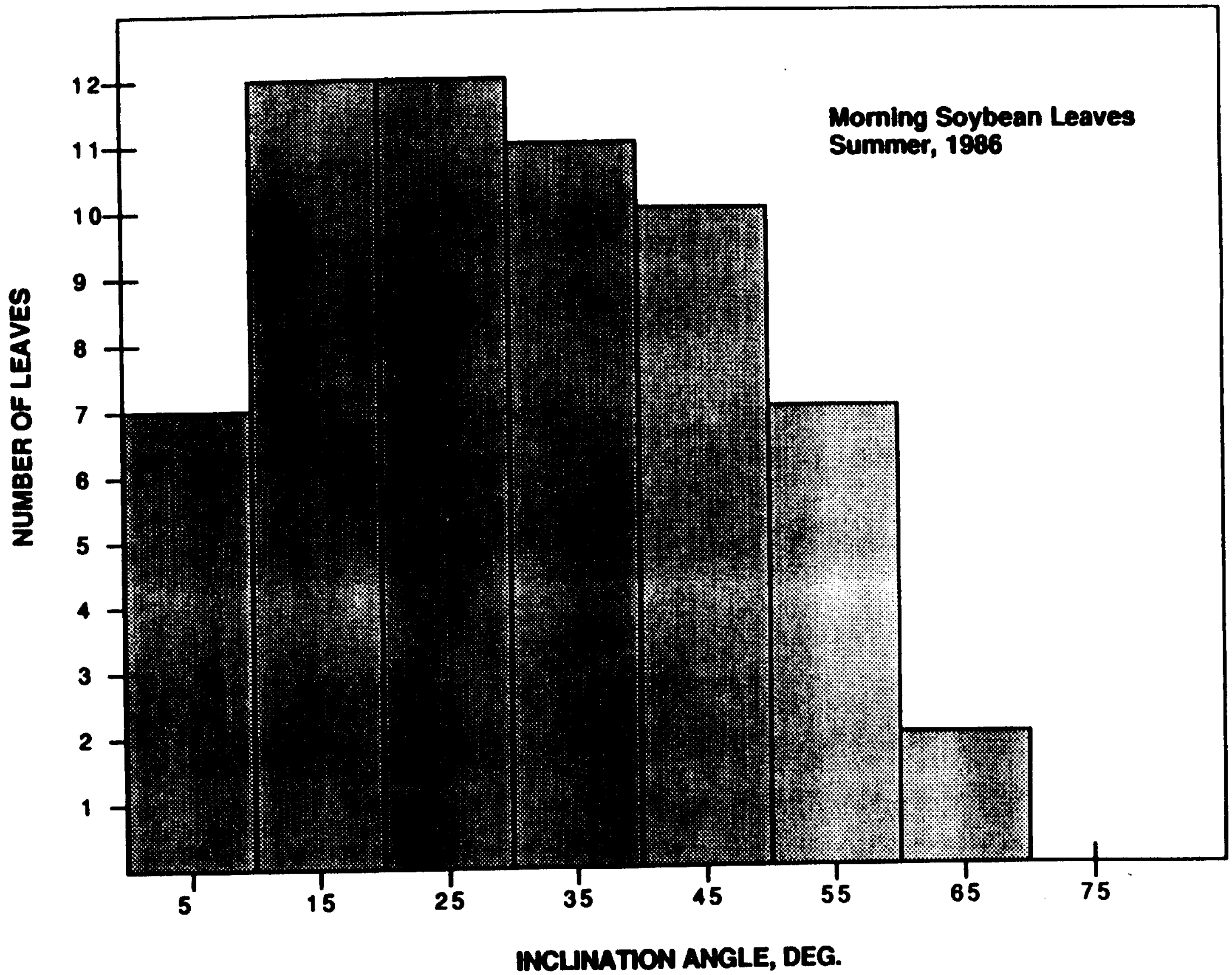


Fig. 3 Leaf Inclination Angle Histogram For Morning.

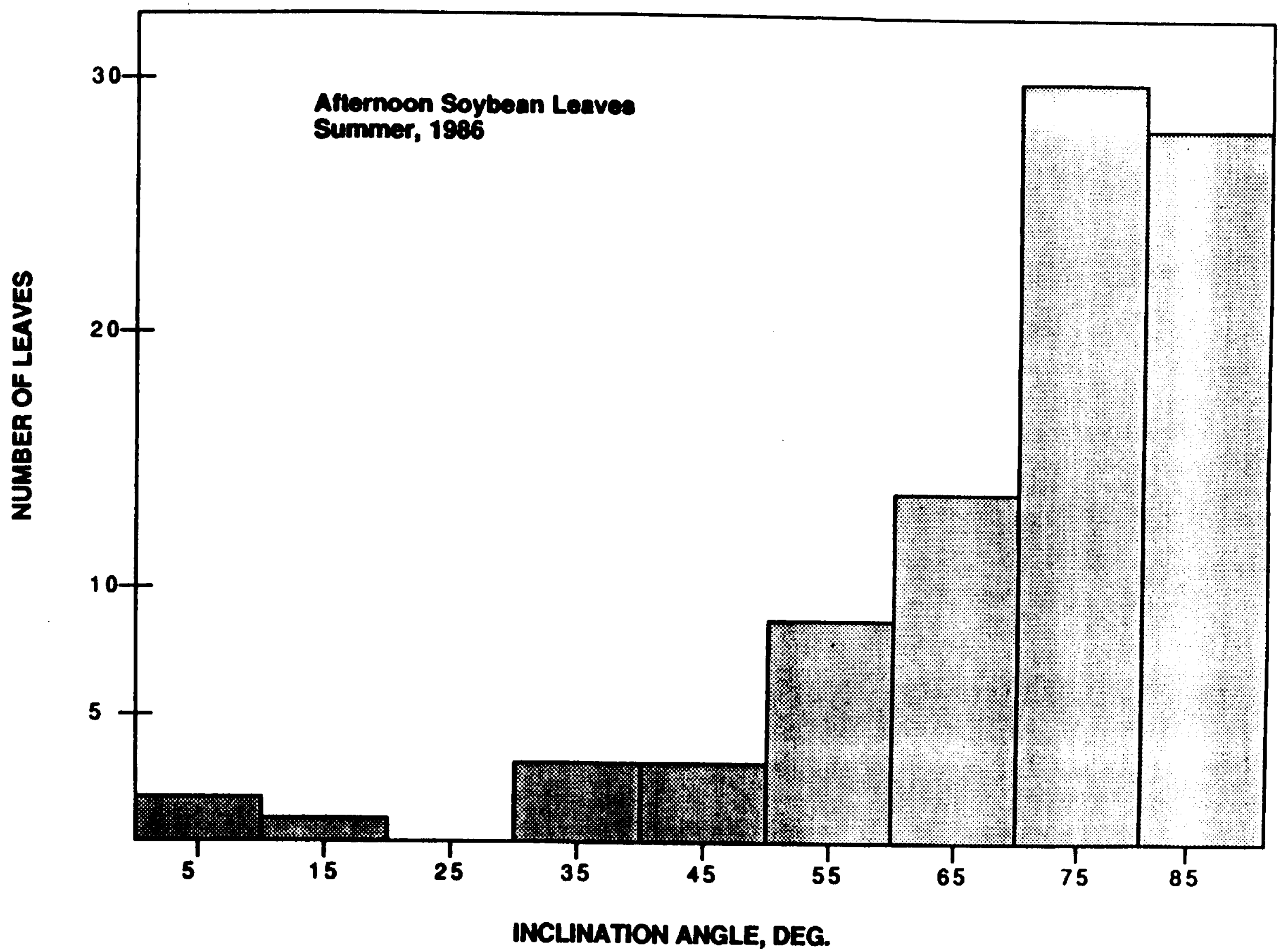


Fig. 4 Leaf Inclination Angle Histogram For Afternoon.

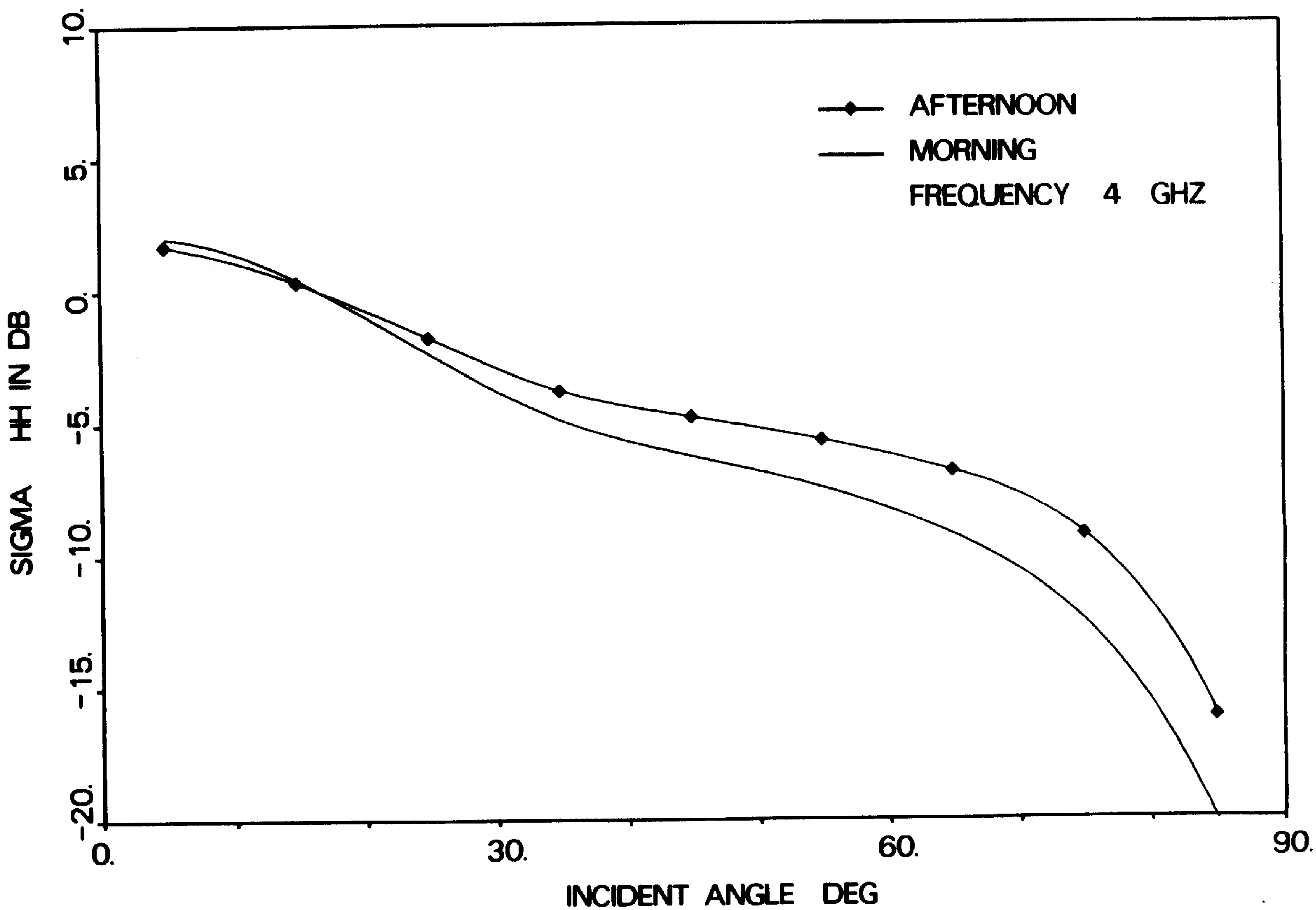


Fig. 5 Backscattering Coefficient vs. Incident Angle For Horizontal Polarization At 4 GHz.

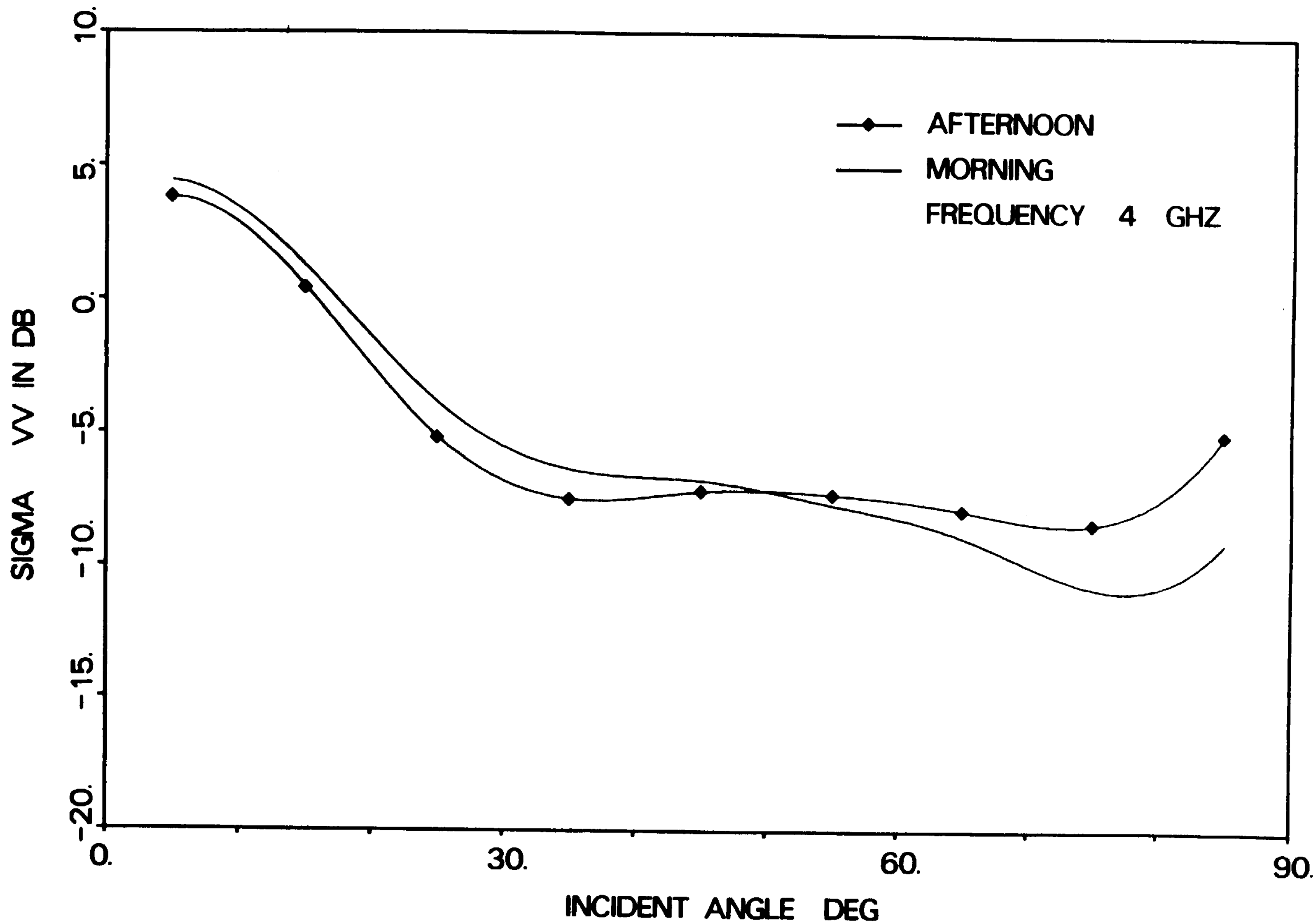


Fig. 6 Backscattering Coefficient vs. Incident Angle For Vertical Polarization At 4 GHz.

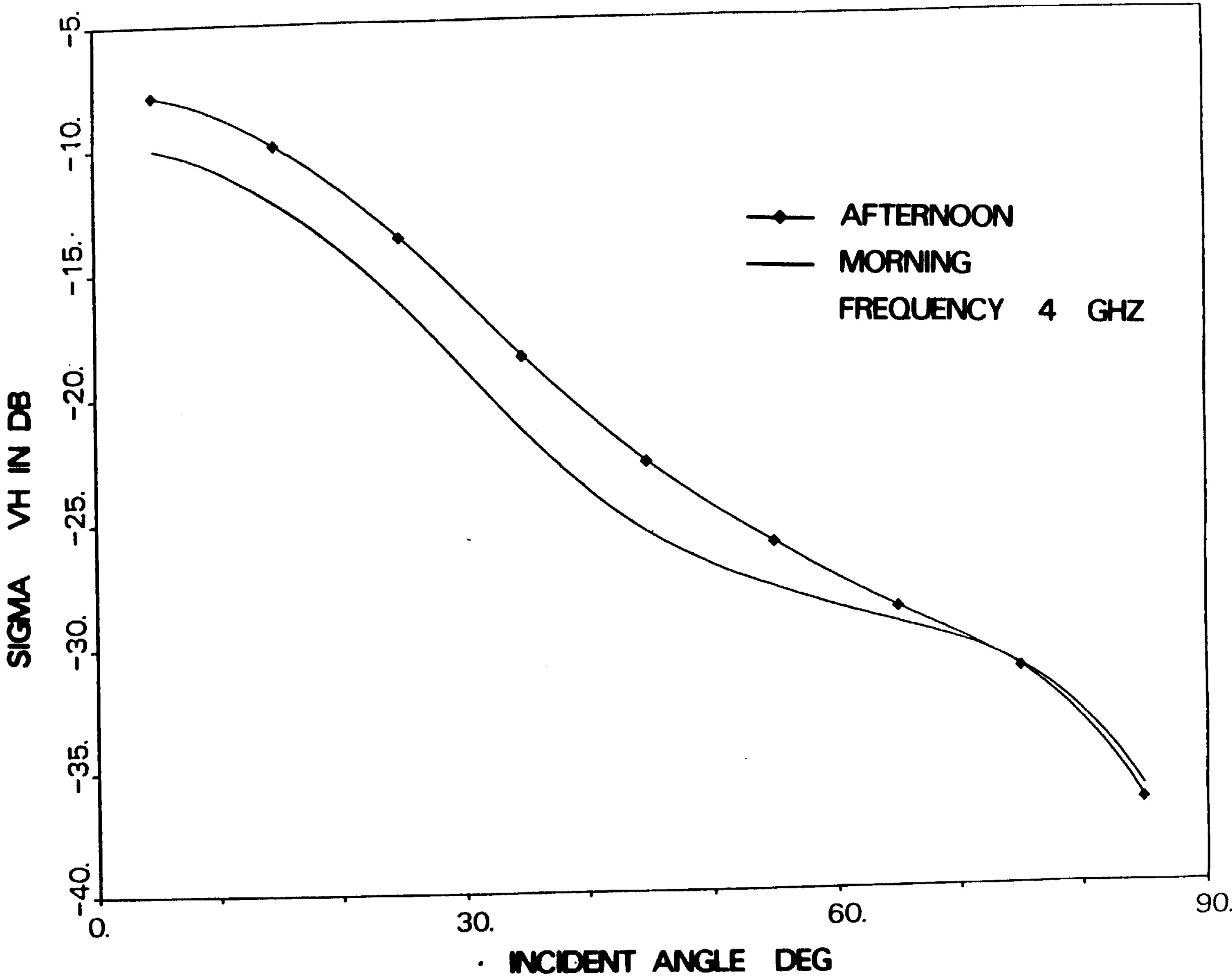


Fig. 7 **Backscattering Coefficient vs. Incident Angle For Cross Polarization At 4 GHz.**

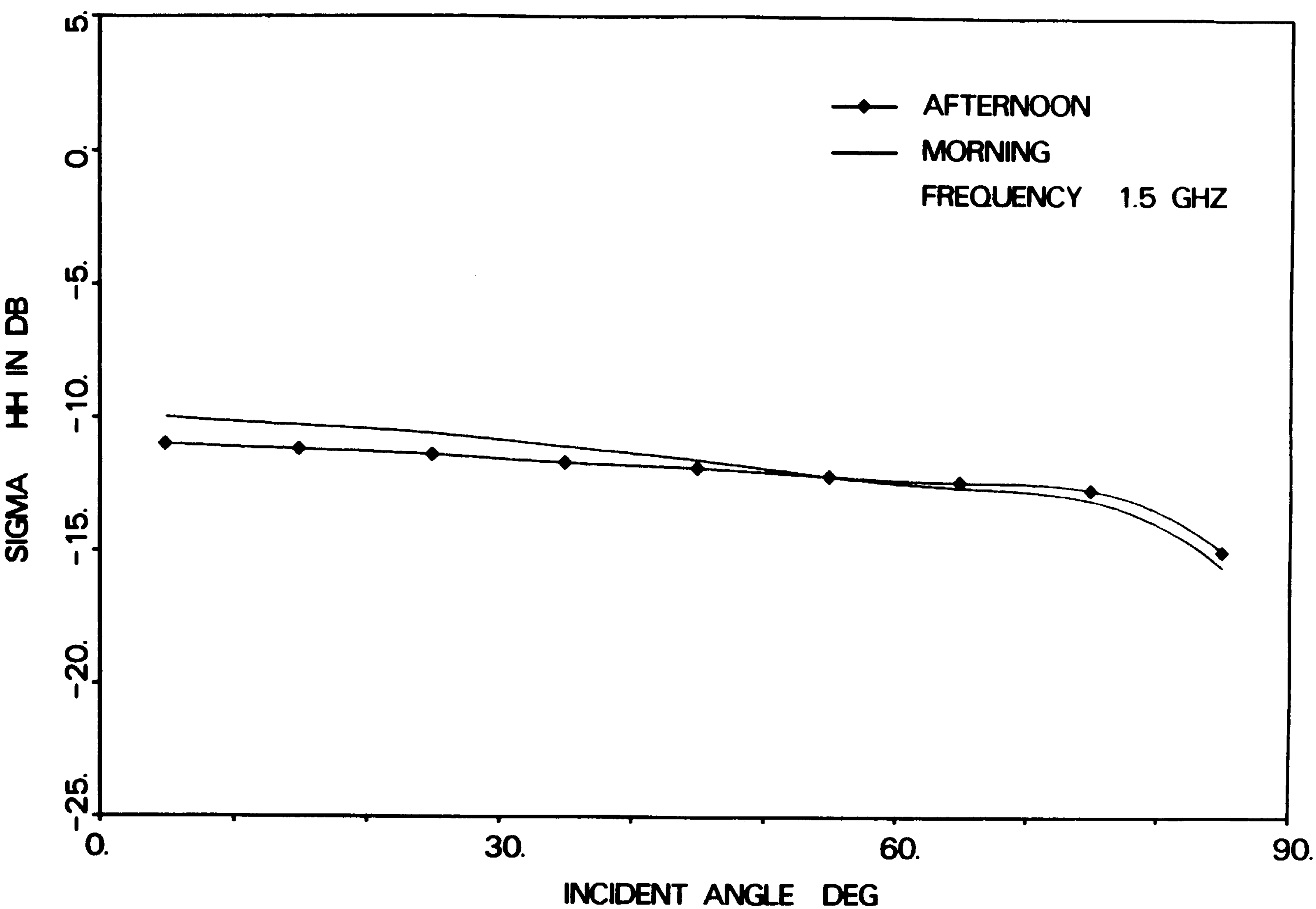


Fig. 8 **Backscattering Coefficient vs. Incident Angle For Horizontal Polarization at 1.5 GHz.**

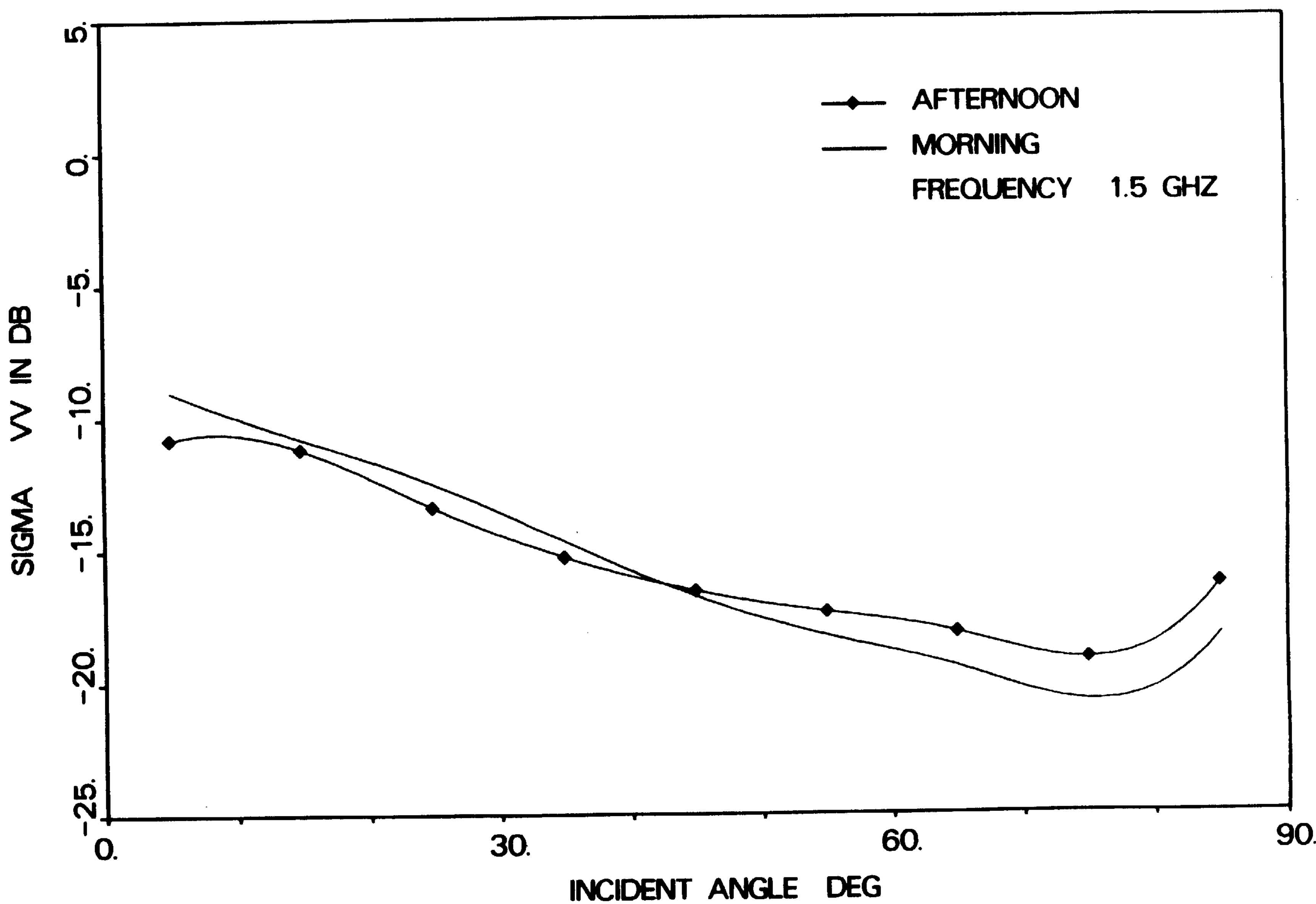


Fig. 9 **Backscattering coefficient vs. Incident Angle For Vertical Polarization At 1.5 GHz.**

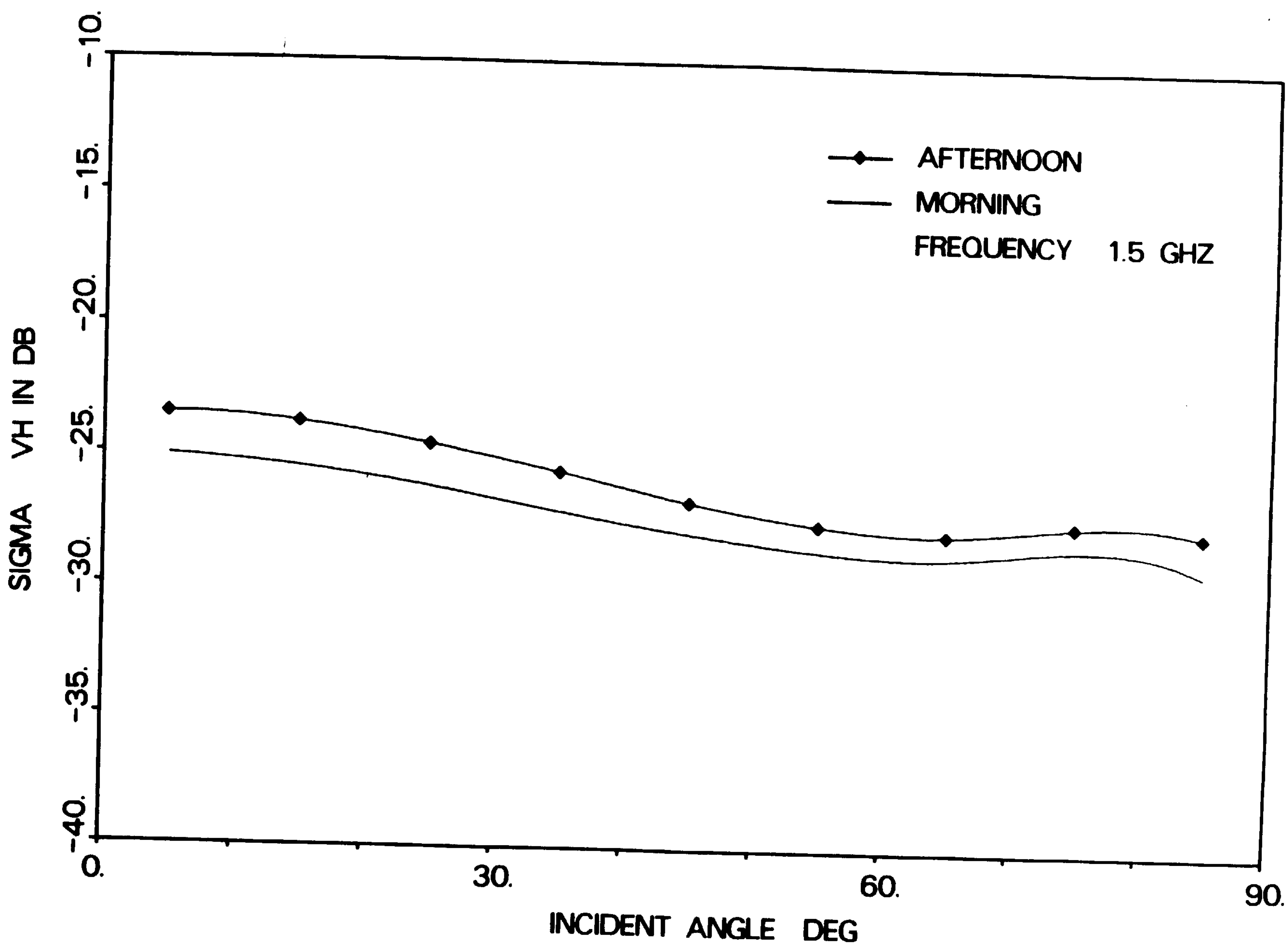


Fig. 10 Backscattering Coefficient vs. Incident Angle For Cross Polarization At 1.5 GHz.

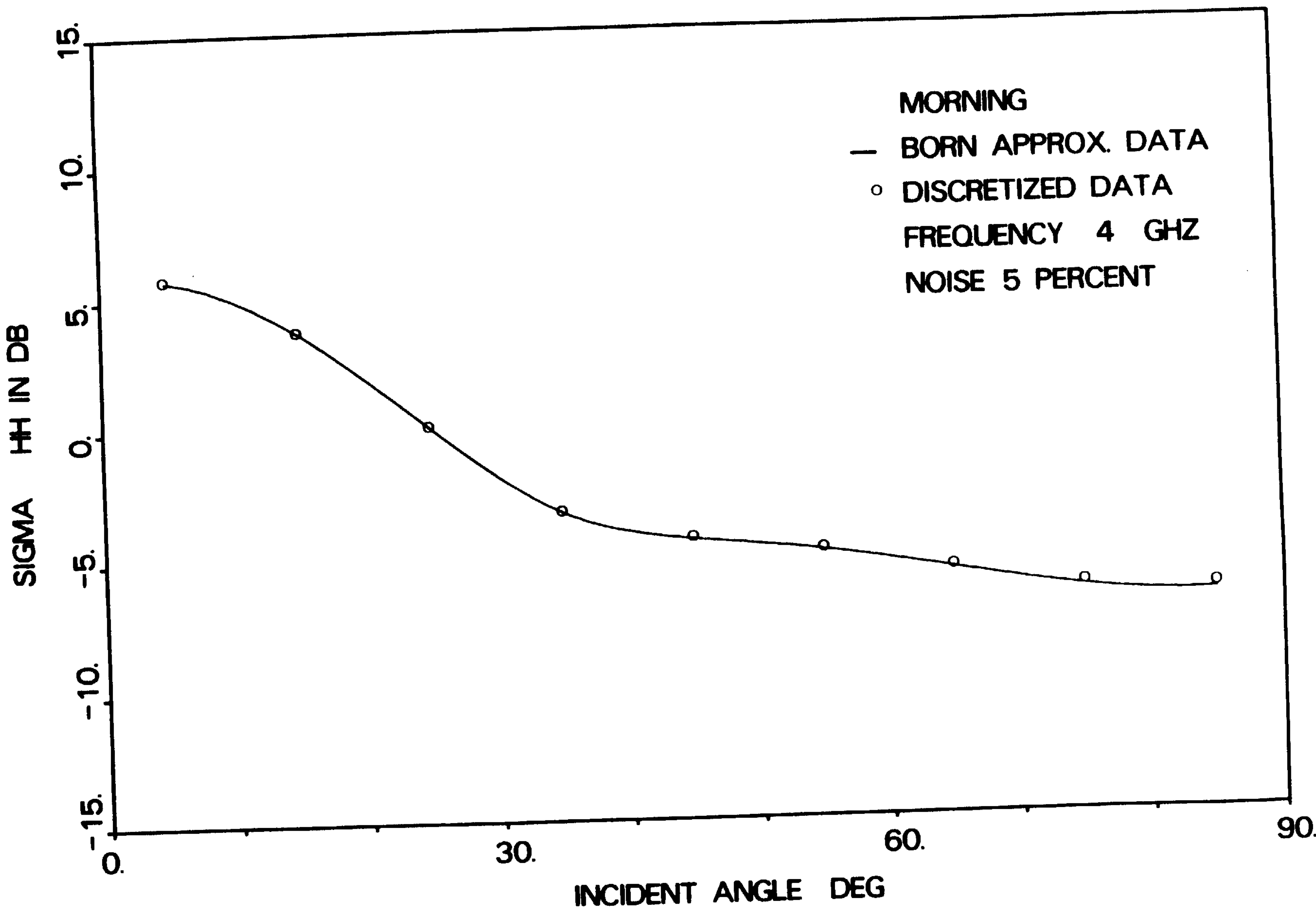


Fig. 11 Morning Backscattering Coefficients vs. Incident Angle For Horizontal Polarization.

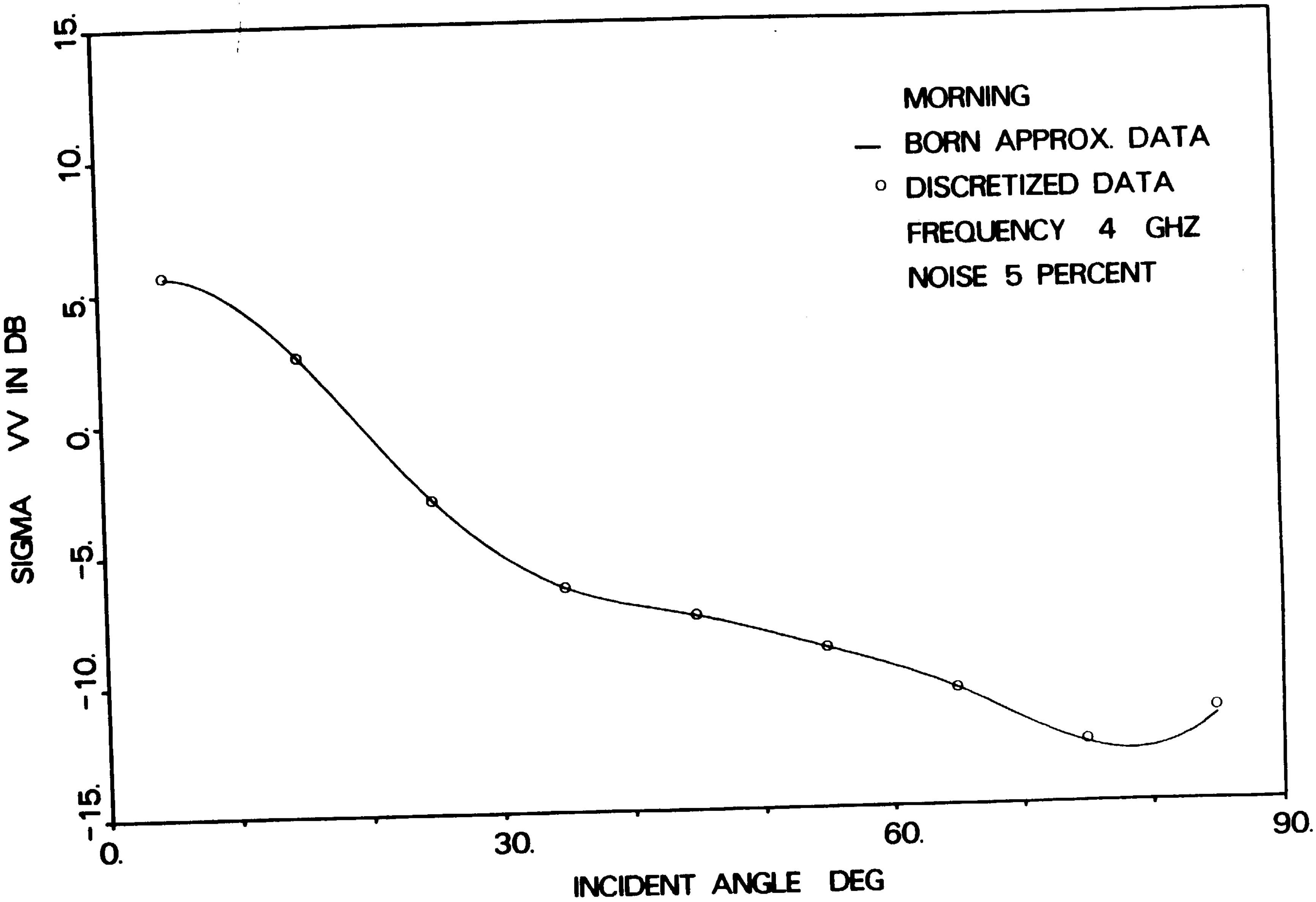


Fig. 12 Morning Backscattering coefficients vs. Incident Angle For Vertical Polarization.

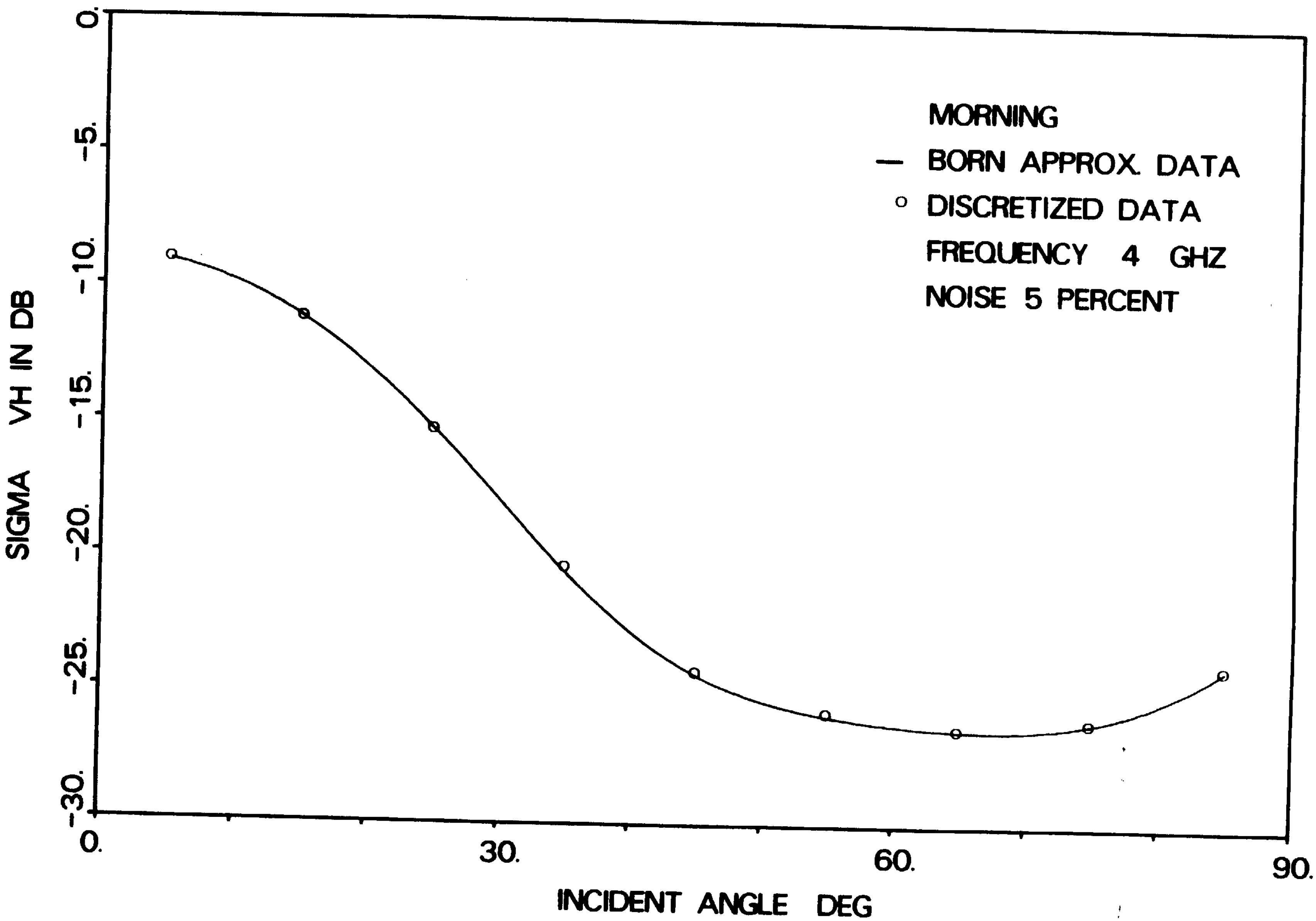


Fig. 13 Morning Backscattering Coefficients vs. Incident Angle For Cross Polarization.

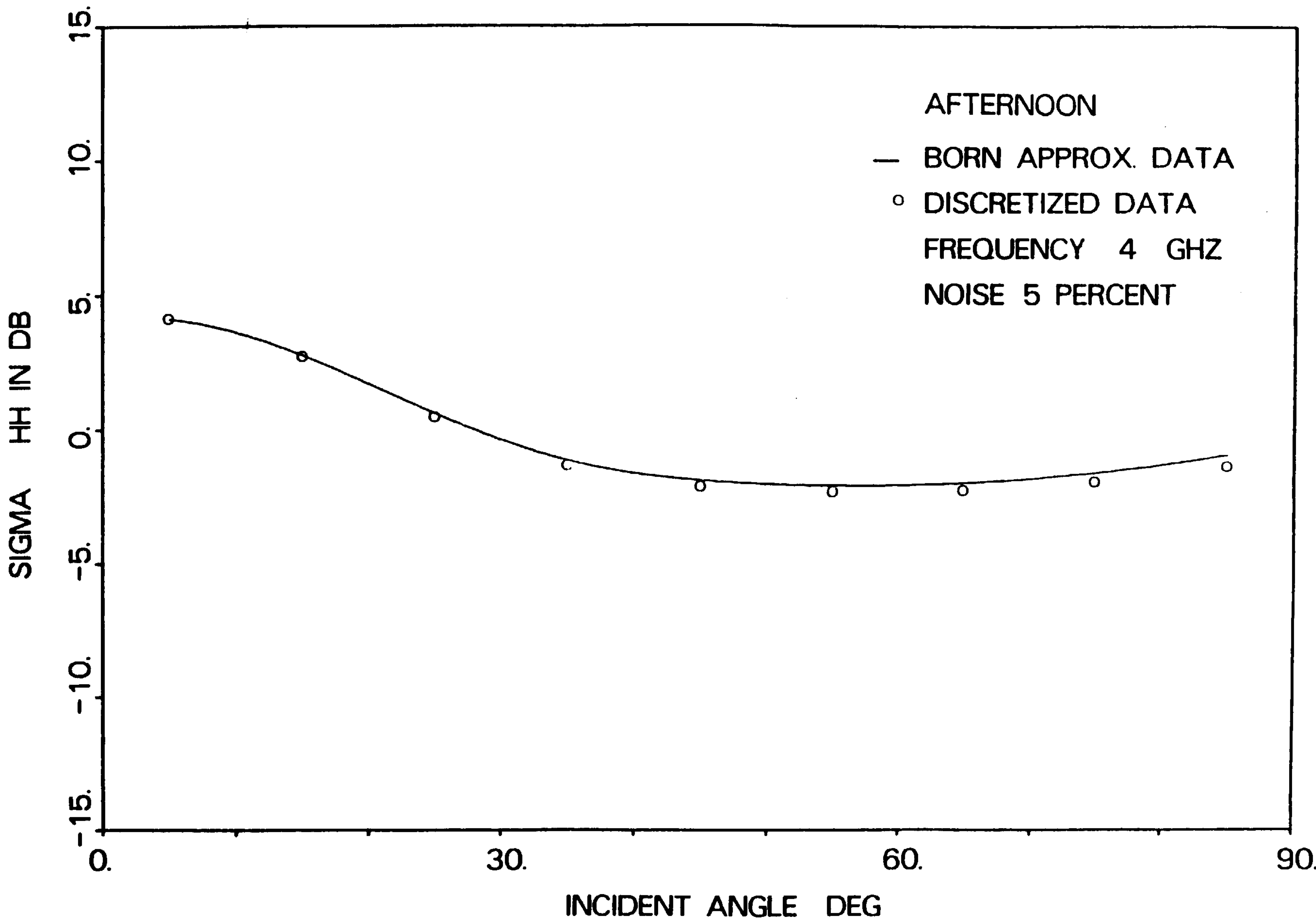


Fig. 14 Afternoon Backscattering Coefficients vs. Incident Angle For Horizontal Polarization.

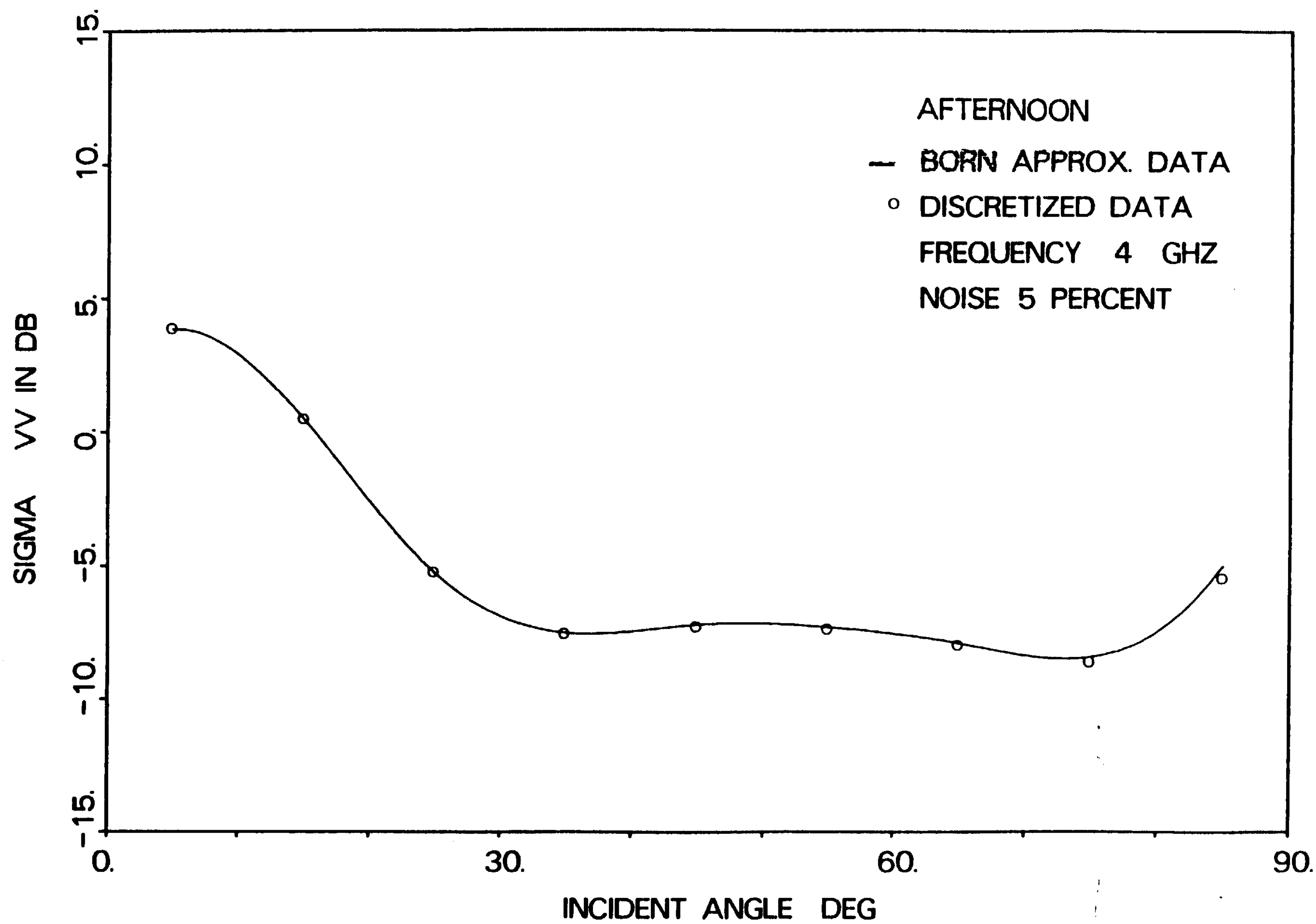


Fig. 15 Afternoon Backscattering Coefficients vs. Incident Angle For Vertical Polarization.

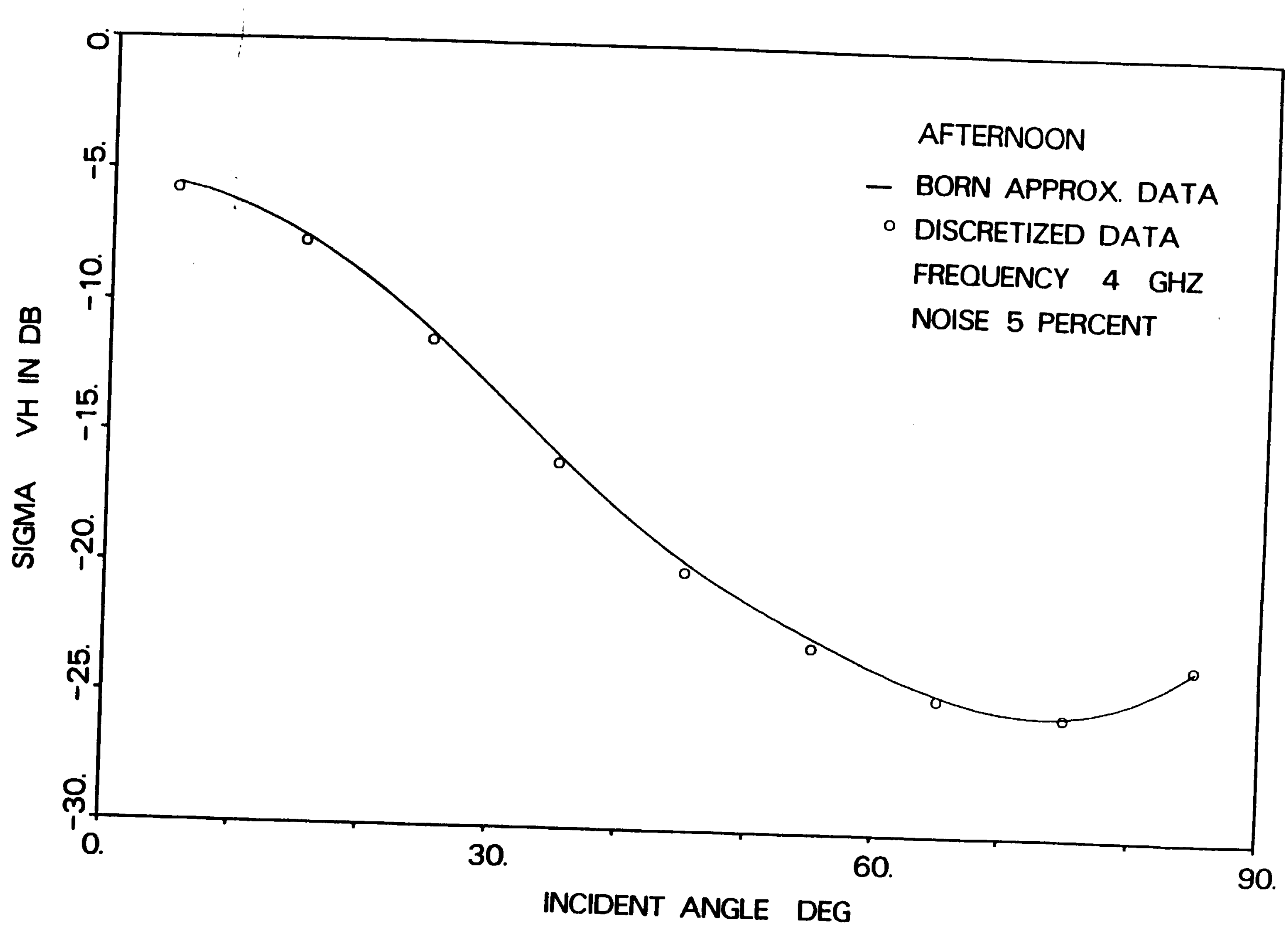


Fig. 16 Afternoon Backscattering Coefficients vs. Incident Angle For Cross-Polarization.

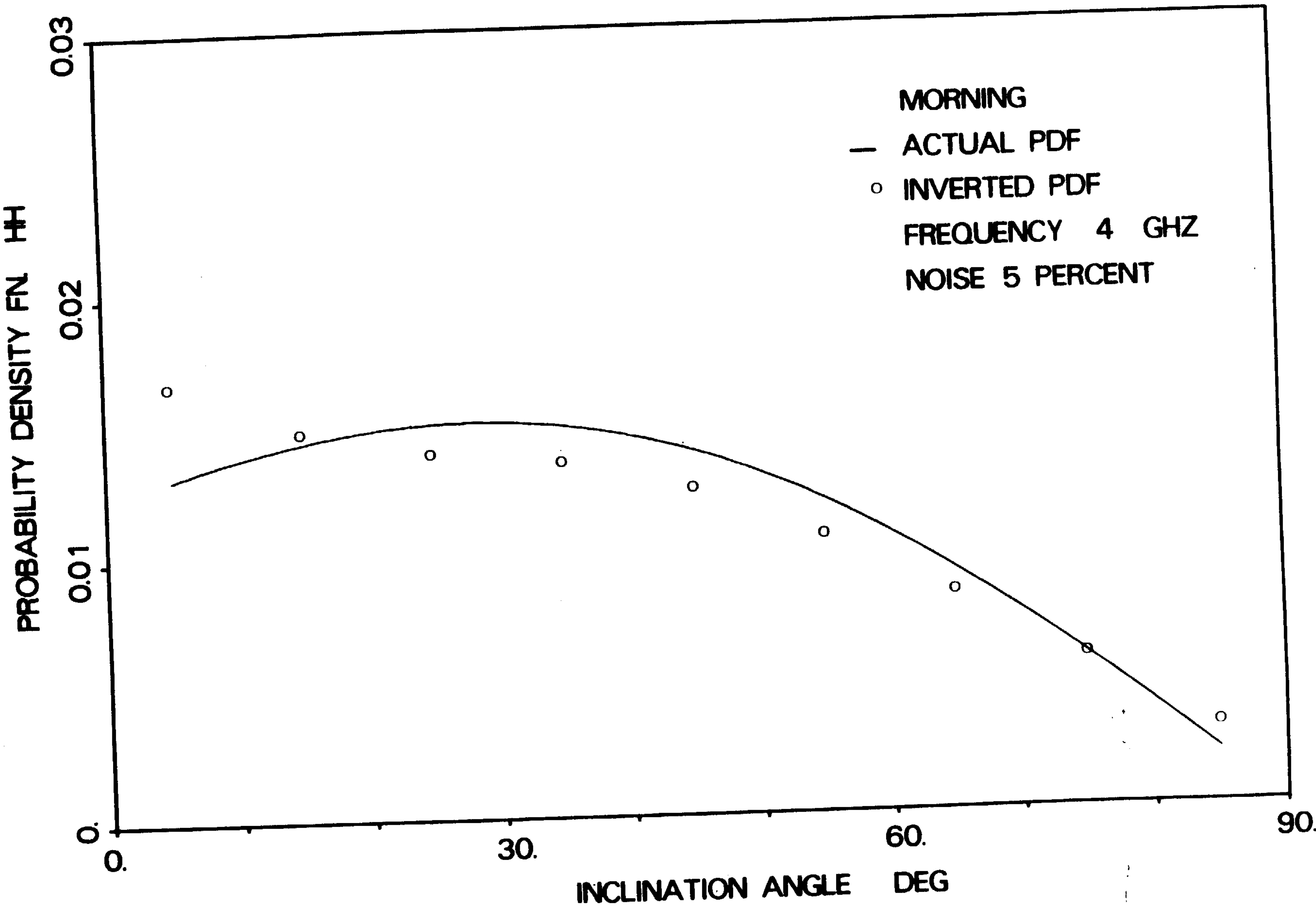


Fig. 17 Probability Density Function For Morning vs. Inclination Angle For Horizontal Polarization - Nine Point Case.

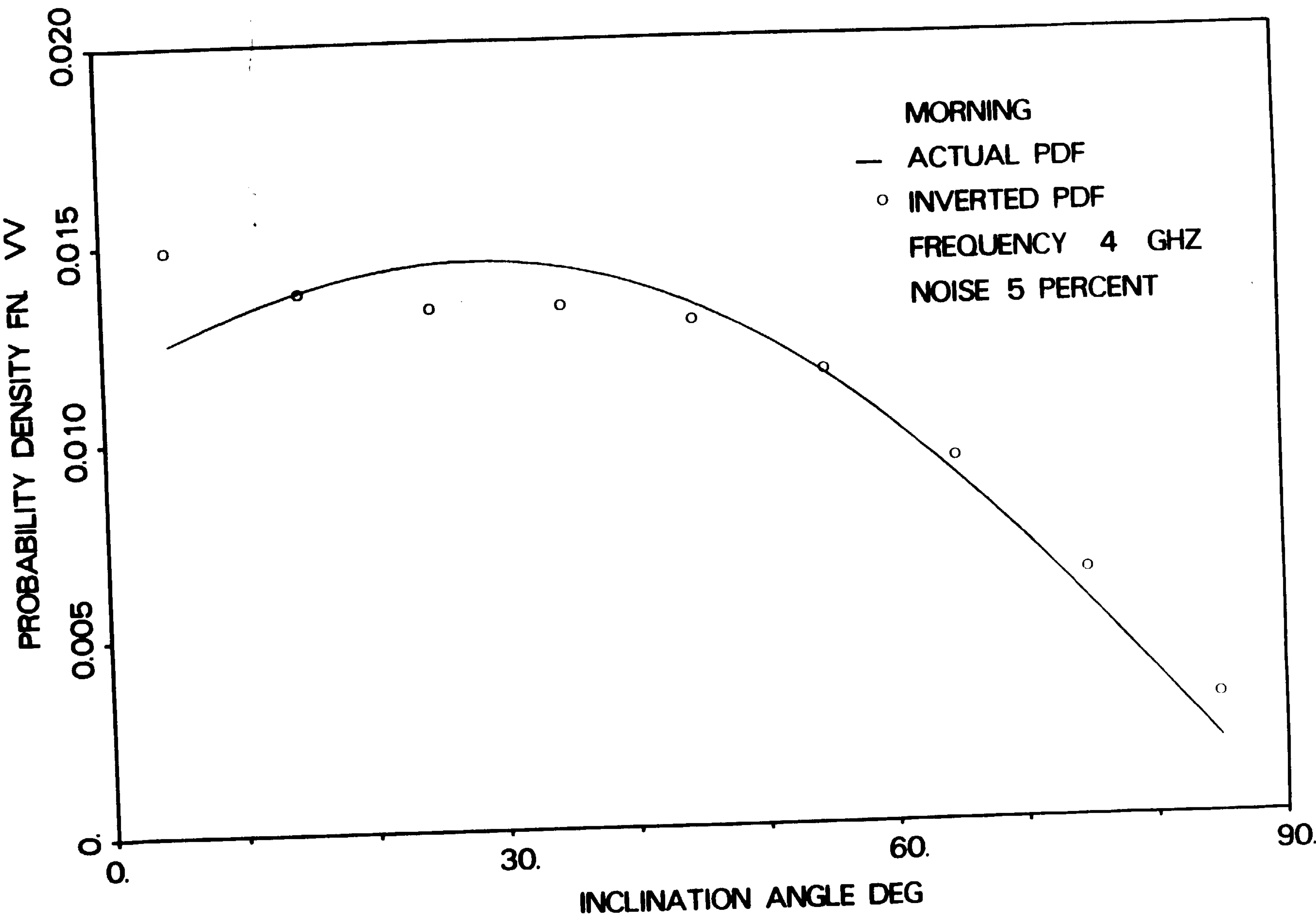


Fig. 18 Probability Density Function For Morning vs. Inclination Angle For Vertical Polarization - Nine Point Case.

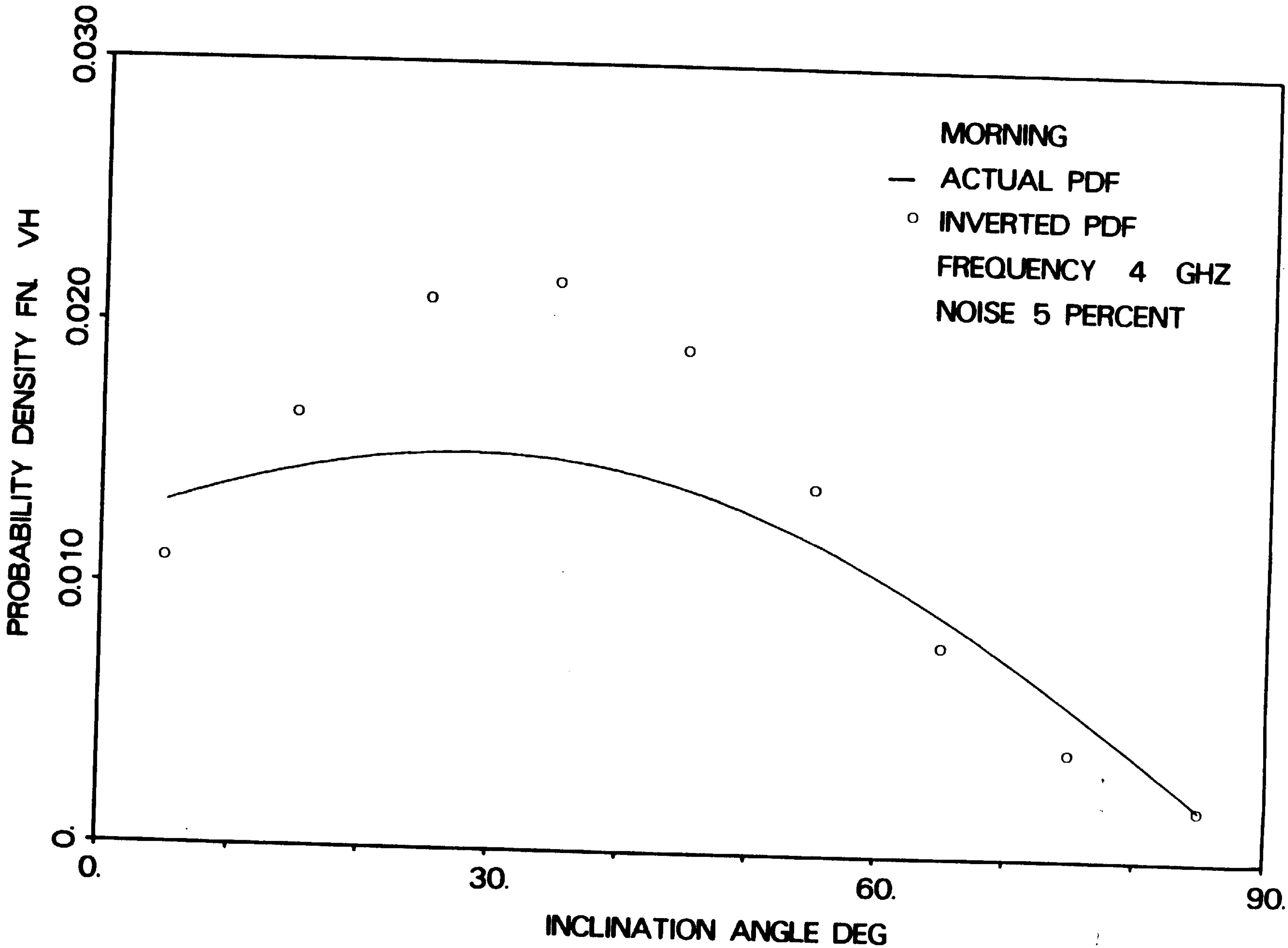


Fig. 19 Probability Density Function For Morning vs. Inclination Angle For Cross Polarization - Nine Point Case.

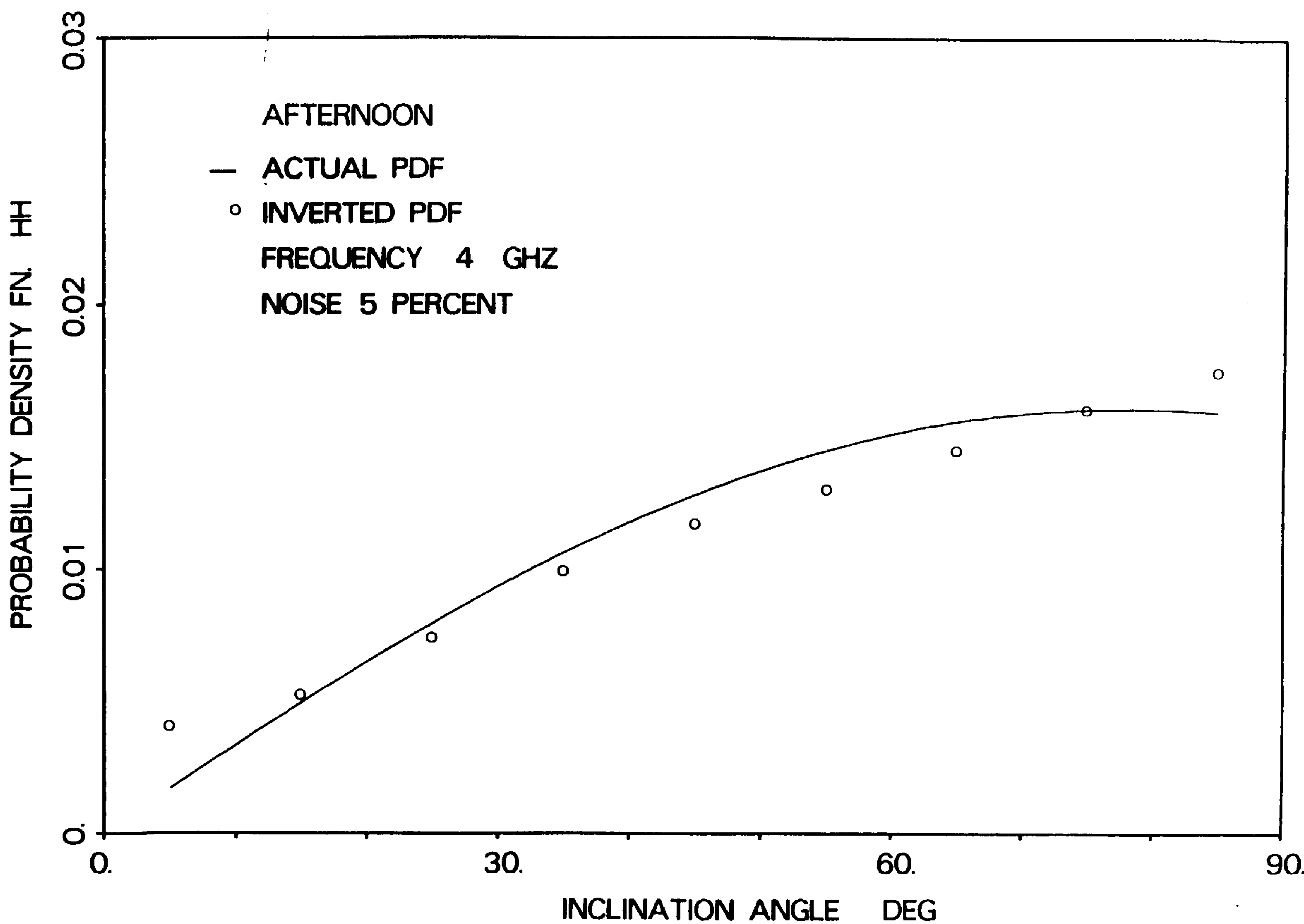


Fig. 20 Probability Density Function For Afternoon vs. Inclination Angle For Horizontal Polarization - Nine Point Case.

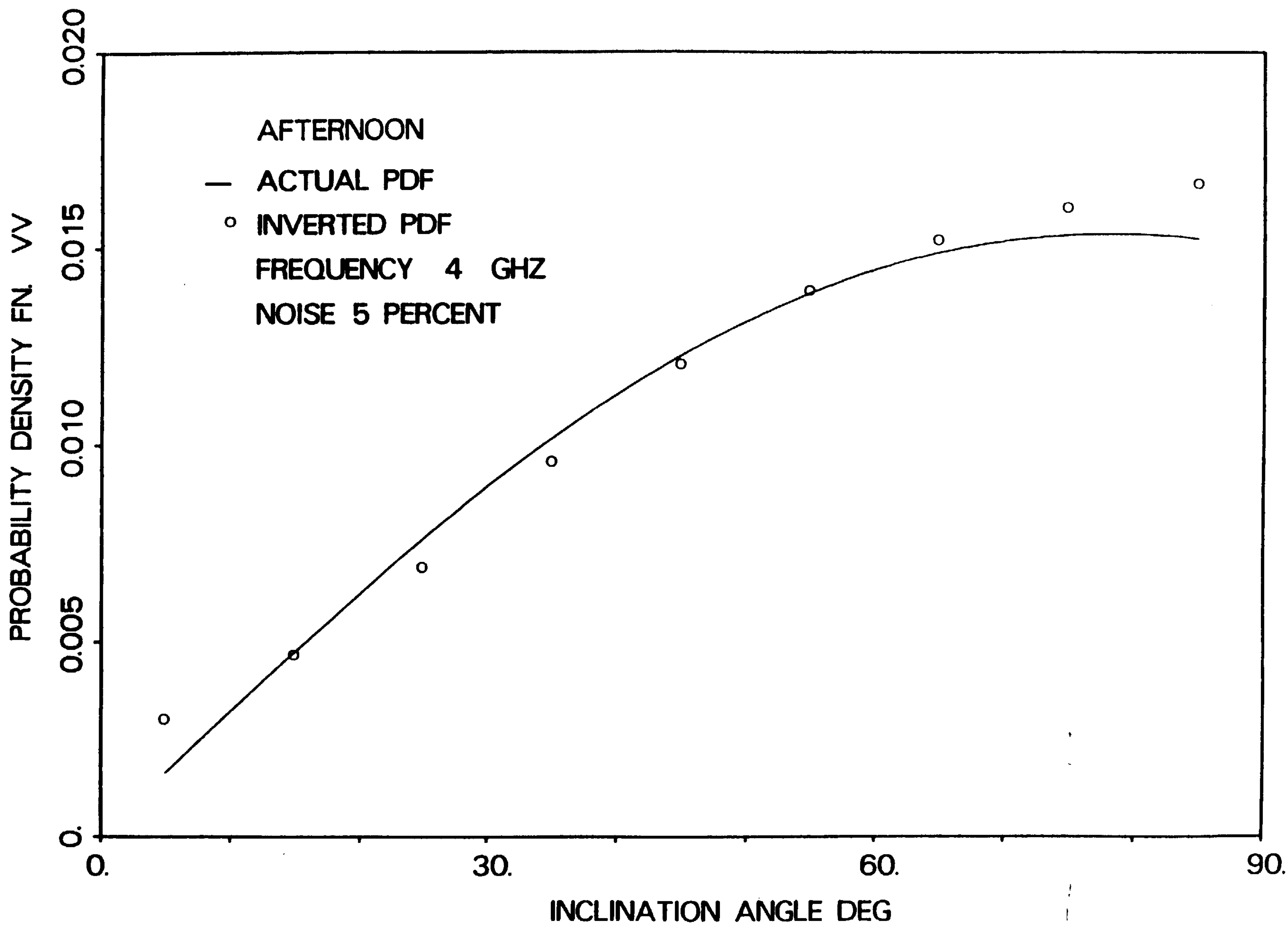


Fig. 21 Probability Density Function for Afternoon vs. Inclination Angle For Vertical Polarization - Nine Point Case.

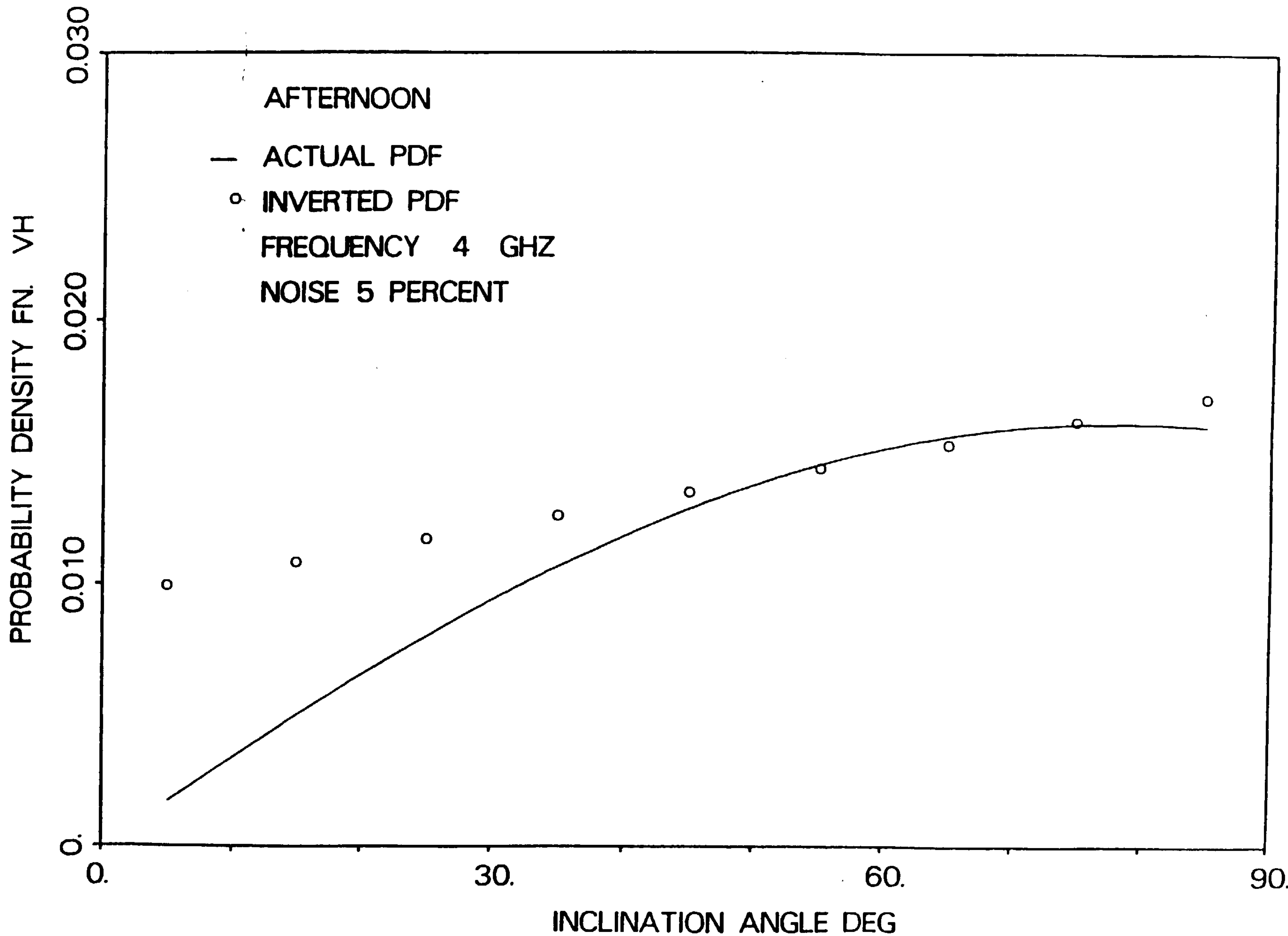


Fig. 22 Probability Density Function For Afternoon vs. Inclination Angle For Cross Polarization - Nine Point Case.

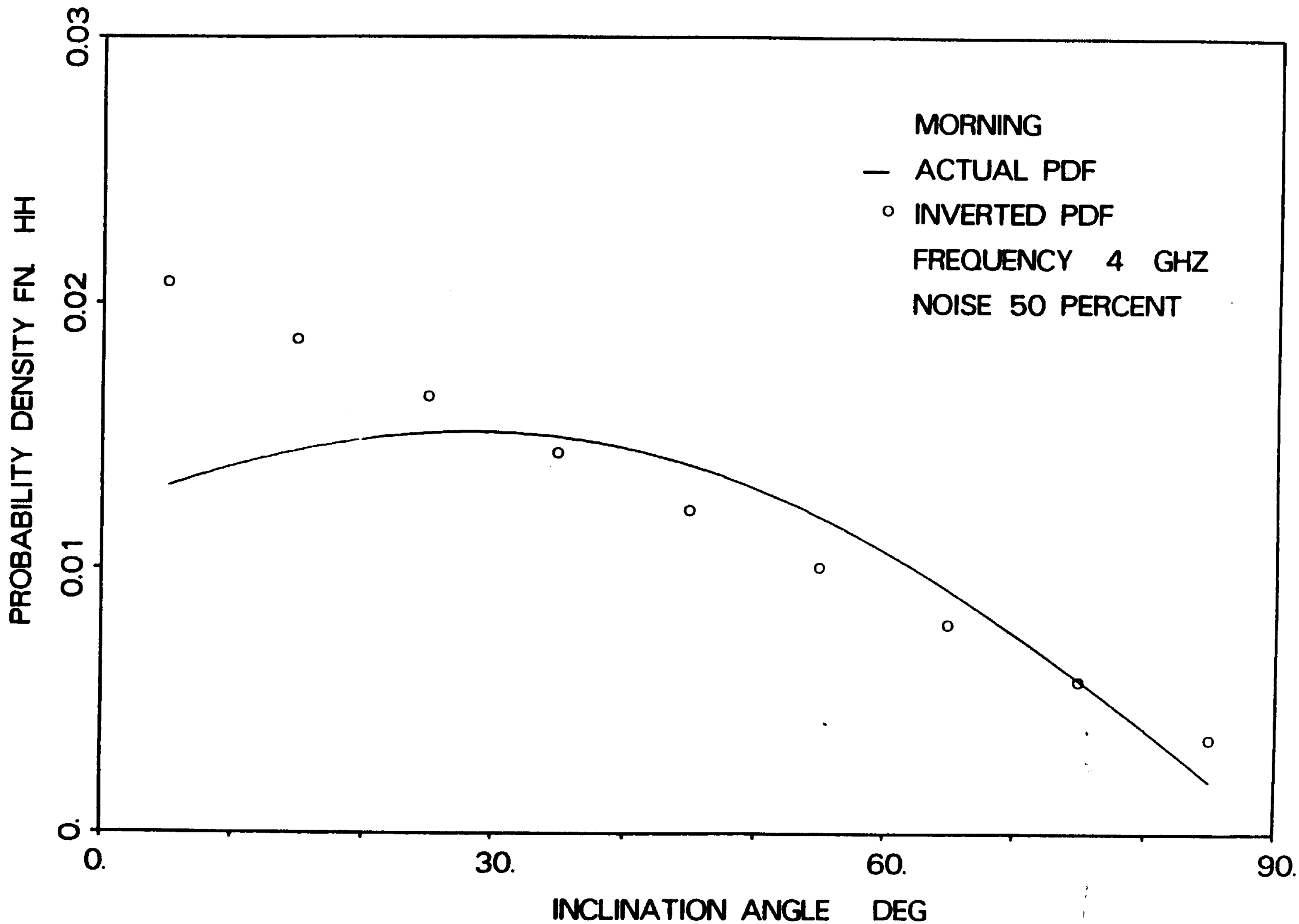


Fig. 23 Probability Density Function For Morning vs. Inclination Angle For Horizontal Polarization with 50 Percent Noise - Nine Point Case.

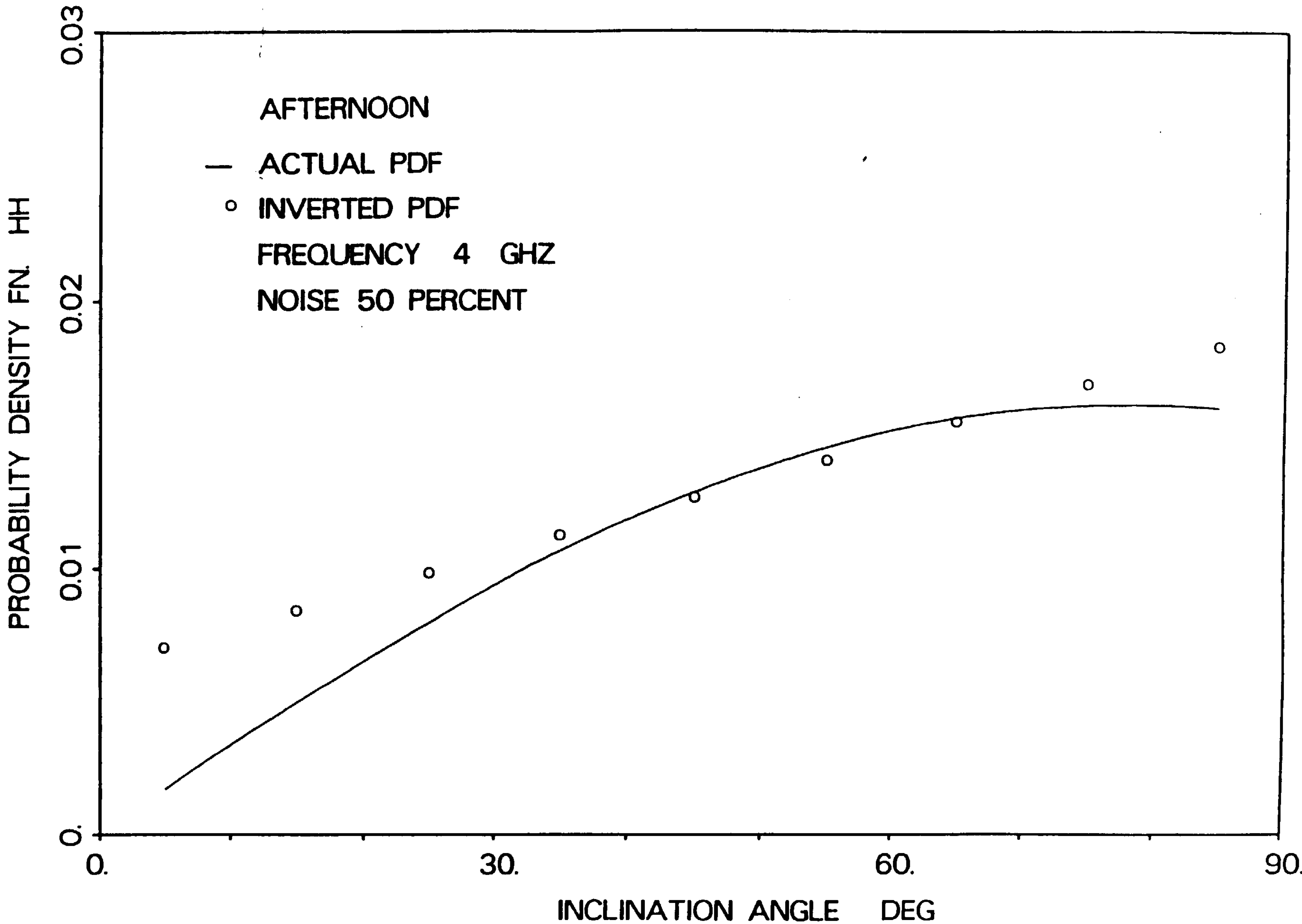


Fig. 24 Probability Density Function For Afternoon vs. Inclination Angle For Horizontal Polarization with 50 Percent Noise - Nine Point Case.

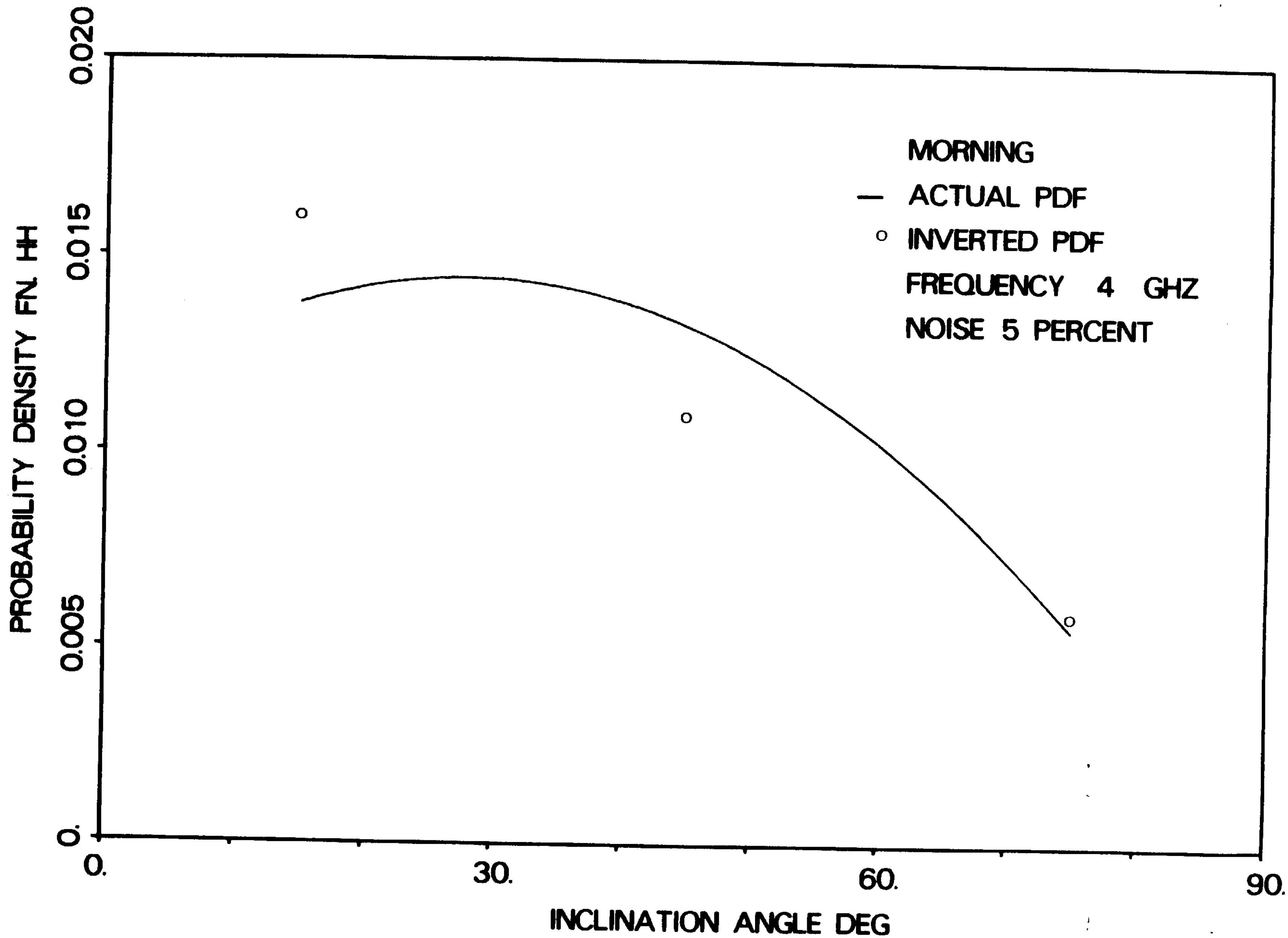


Fig. 25 Probability Density Function For Morning vs. Inclination Angle For Horizontal Polarization - Three Point Case.

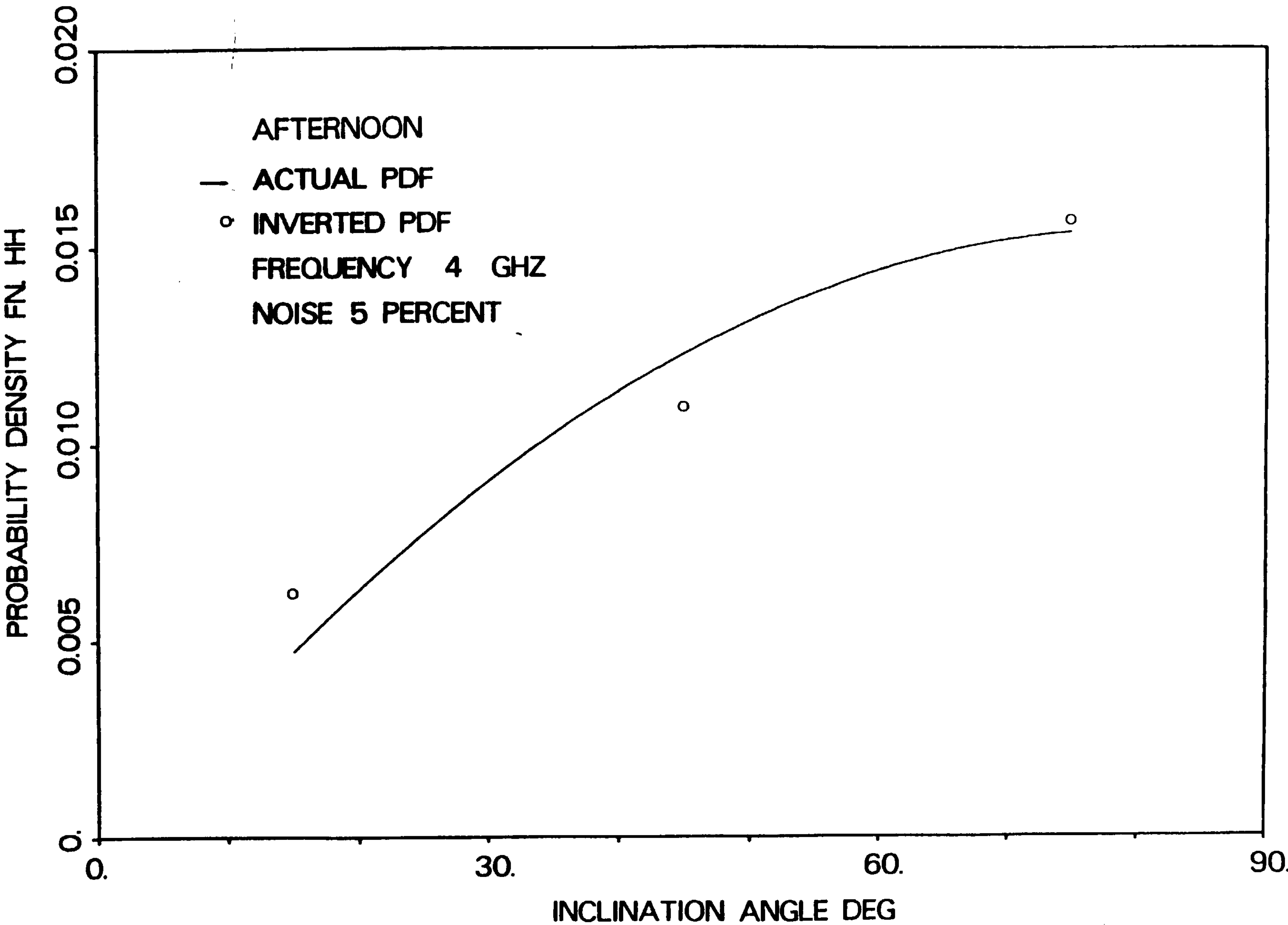


Fig. 26 Probability Density Function For Afternoon vs. Inclination Angle For Horizonal Polarization - Three Point Case.

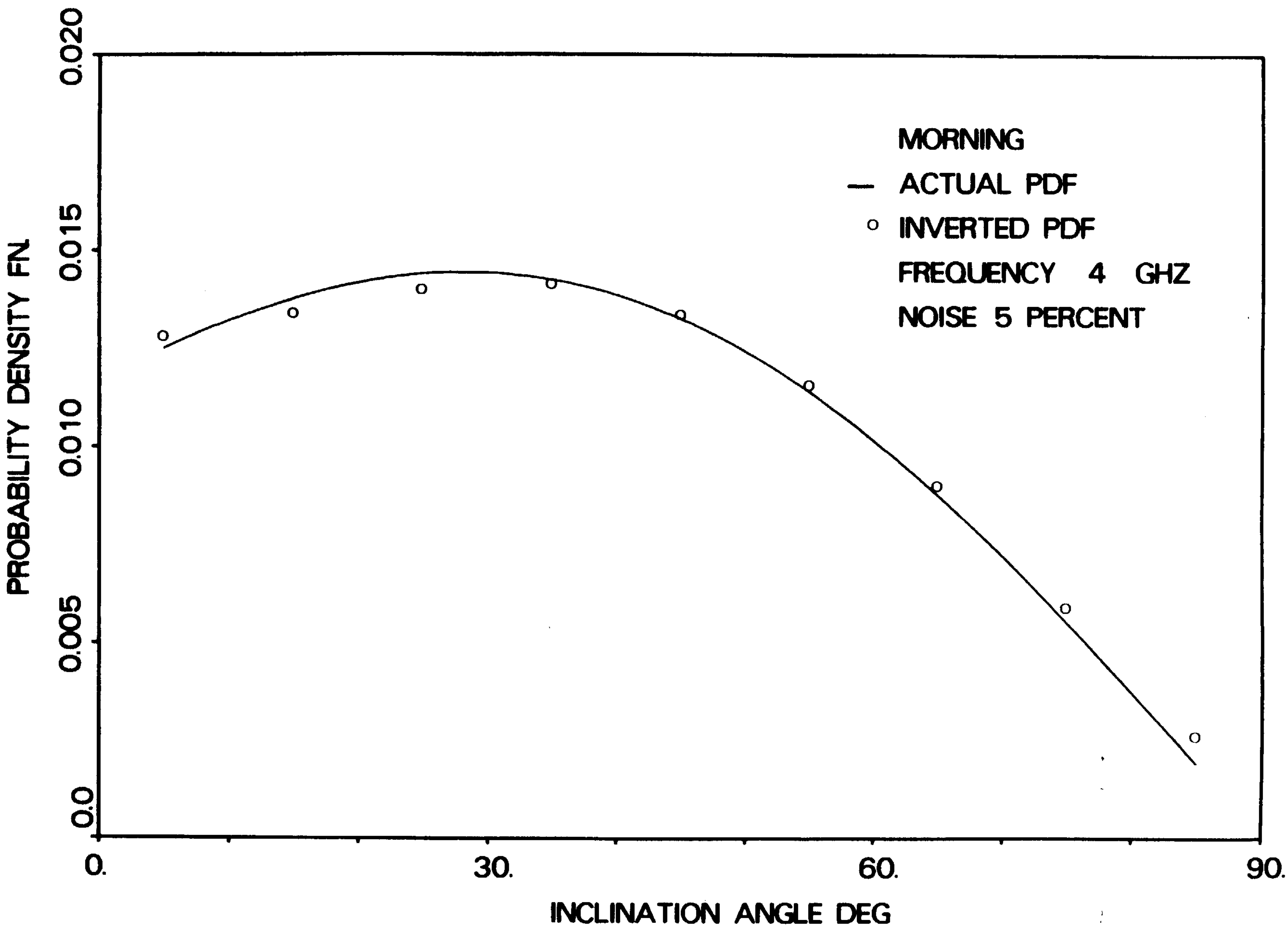


Fig. 27 Probability Density Function For Morning vs. Inclination Angle For Combined Polarization.

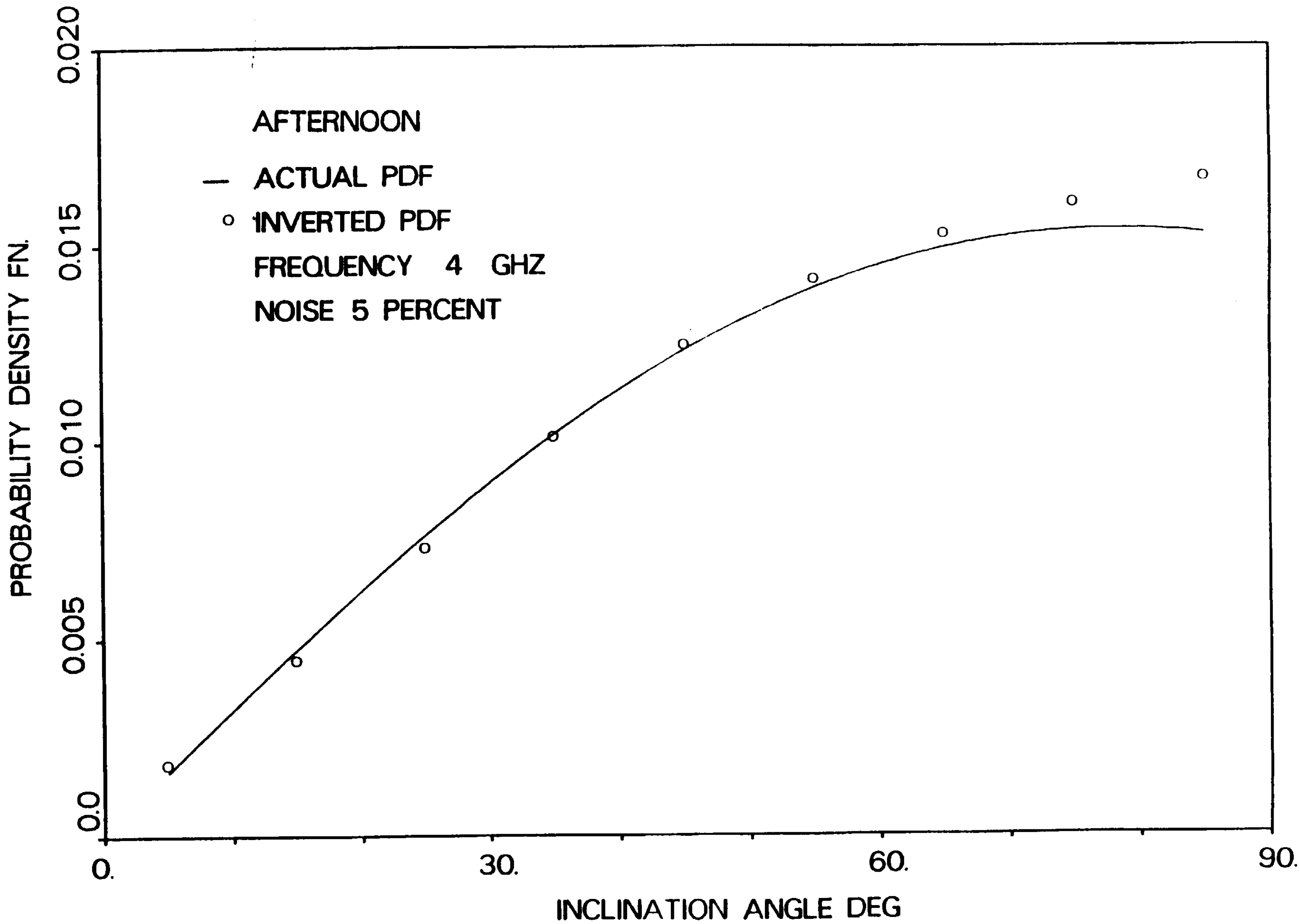


Fig. 28 Probability Density Function For Afternoon vs. Inclination Angle For Combined Polarization.

APPENDIX A

SOYBEAN LEAF DATA

Following acronyms have been used.

MO - Month

DY - Hour

HR - Hour

MN - Minutes

ST - Stem Number

LF - Leaflet Number

LFT - Leaf Inclination Angle
(Measured From the Vertical)

MO	DY	HR	MN	ST	LF	LFT
8	18	19	36	1	1	120
8	18	19	36	1	2	105
8	18	19	36	1	3	120
8	18	19	36	2	1	105
8	18	19	36	2	2	90
8	18	19	36	2	3	95
8	18	19	36	3	1	80
8	18	19	36	3	2	105
8	18	19	36	3	3	65
8	18	19	36	4	1	112
8	18	19	36	4	2	75
8	18	19	36	4	3	90
8	18	19	36	5	1	120
8	18	19	36	5	2	140
8	18	19	36	5	3	120
8	18	19	36	6	1	85
8	18	19	36	6	2	125
8	18	19	36	6	3	60
8	18	20	58	1	1	135
8	18	20	58	1	2	120
8	18	20	58	1	3	125
8	18	20	58	2	1	125
8	18	20	58	2	2	118
8	18	20	58	2	3	120
8	18	20	58	3	1	110
8	18	20	58	3	2	135
8	18	20	58	3	3	110
8	18	20	58	4	1	142
8	18	20	58	4	2	135
8	18	20	58	4	3	165
8	18	20	58	5	1	128
8	18	20	58	5	2	162
8	18	20	58	5	3	112
8	18	20	58	6	1	122
8	18	20	58	6	2	142
8	18	20	58	6	3	120
8	18	22	00	1	1	170
8	18	22	00	1	2	135
8	18	22	00	1	3	138
8	18	22	00	2	1	138
8	18	22	00	2	2	125
8	18	22	00	2	3	120
8	18	22	00	3	1	158
8	18	22	00	3	2	150
8	18	22	00	3	3	123
8	18	22	00	4	1	175
8	18	22	00	4	2	160
8	18	22	00	4	3	170
8	18	22	00	5	1	165
8	18	22	00	5	2	190
8	18	22	00	5	3	165

8	18	22	00	6	1	160
8	18	22	00	6	2	165
8	18	22	00	6	3	150
8	18	22	59	1	1	160
8	18	22	59	1	2	140
8	18	22	59	1	3	120
8	18	22	59	2	1	160
8	18	22	59	2	2	125
8	18	22	59	2	3	135
8	18	22	59	3	1	145
8	18	22	59	3	2	110
8	18	22	59	3	3	135
8	18	22	59	4	1	170
8	18	22	59	4	2	180
8	18	22	59	4	3	160
8	18	22	59	5	1	150
8	18	22	59	5	2	160
8	18	22	59	5	3	160
8	18	22	59	6	1	150
8	18	22	59	6	2	155
8	18	22	59	6	3	155
8	18	23	55	1	1	135
8	18	23	55	1	2	120
8	18	23	55	1	3	130
8	18	23	55	2	1	160
8	18	23	55	2	2	155
8	18	23	55	2	3	135
8	18	23	55	3	1	145
8	18	23	55	3	2	140
8	18	23	55	3	3	135
8	18	23	55	4	1	180
8	18	23	55	4	2	180
8	18	23	55	4	3	175
8	18	23	55	5	1	150
8	18	23	55	5	2	140
8	18	23	55	5	3	155
8	18	23	55	6	1	150
8	18	23	55	6	2	165
8	18	23	55	6	3	140
8	19	1	12	1	1	172
8	19	1	12	1	2	168
8	19	1	12	1	3	170
8	19	1	12	2	1	999
8	19	1	12	2	2	999
8	19	1	12	2	3	999
8	19	1	12	3	1	999
8	19	1	12	3	2	999
8	19	1	12	3	3	999
8	19	1	12	4	1	999
8	19	1	12	4	2	999
8	19	1	12	4	3	999

8	19	1	12	5	1	170
8	19	1	12	5	2	170
8	19	1	12	5	3	160
8	19	1	12	6	1	999
8	19	1	12	6	2	999
8	19	1	12	6	3	999
8	19	2	30	1	1	175
8	19	2	30	1	2	168
8	19	2	30	1	3	175
8	19	2	30	2	1	999
8	19	2	30	2	2	999
8	19	2	30	2	3	999
8	19	2	30	3	1	168
8	19	2	30	3	2	165
8	19	2	30	3	3	160
8	19	2	30	4	1	999
8	19	2	30	4	2	999
8	19	2	30	4	3	999
8	19	2	30	5	1	170
8	19	2	30	5	2	165
8	19	2	30	5	3	170
8	19	2	30	6	1	999
8	19	2	30	6	2	999
8	19	2	30	6	3	999
8	19	4	15	1	1	185
8	19	4	15	1	2	170
8	19	4	15	1	3	180
8	19	4	15	2	1	999
8	19	4	15	2	2	999
8	19	4	15	2	3	999
8	19	4	15	3	1	999
8	19	4	15	3	2	999
8	19	4	15	3	3	999
8	19	4	15	4	1	999
8	19	4	15	4	2	999
8	19	4	15	4	3	999
8	19	4	15	5	1	999
8	19	4	15	5	2	999
8	19	4	15	5	3	999
8	19	4	15	6	1	999
8	19	4	15	6	2	999
8	19	4	15	6	3	999
8	19	5	30	1	1	175
8	19	5	30	1	2	165
8	19	5	30	1	3	185
8	19	5	30	2	1	999
8	19	5	30	2	2	999
8	19	5	30	2	3	999
8	19	5	30	3	1	999
8	19	5	30	3	2	999
8	19	5	30	3	3	999

8	19	5	30	4	1	999
8	19	5	30	4	2	999
8	19	5	30	4	3	999
8	19	5	30	5	1	148
8	19	5	30	5	2	155
8	19	5	30	5	3	155
8	19	5	30	6	1	999
8	19	5	30	6	2	999
8	19	5	30	6	3	999
8	19	6	40	1	1	160
8	19	6	40	1	2	165
8	19	6	40	1	3	182
8	19	6	40	2	1	999
8	19	6	40	2	2	999
8	19	6	40	2	3	999
8	19	6	40	3	1	999
8	19	6	40	3	2	999
8	19	6	40	3	3	999
8	19	6	40	4	1	999
8	19	6	40	4	2	999
8	19	6	40	4	3	999
8	19	6	40	5	1	145
8	19	6	40	5	2	160
8	19	6	40	5	3	168
8	19	6	40	6	1	999
8	19	6	40	6	2	999
8	19	6	40	6	3	999
8	19	7	34	1	1	160
8	19	7	34	1	2	166
8	19	7	34	1	3	180
8	19	7	34	2	1	155
8	19	7	34	2	2	165
8	19	7	34	2	3	160
8	19	7	34	3	1	135
8	19	7	34	3	2	160
8	19	7	34	3	3	160
8	19	7	34	4	1	175
8	19	7	34	4	2	155
8	19	7	34	4	3	155
8	19	7	34	5	1	120
8	19	7	34	5	2	154
8	19	7	34	5	3	175
8	19	7	34	6	1	120
8	19	7	34	6	2	140
8	19	7	34	6	3	150
8	19	8	25	1	1	140
8	19	8	25	1	2	145
8	19	8	25	1	3	160
8	19	8	25	2	1	135
8	19	8	25	2	2	150
8	19	8	25	2	3	135

8	19	8	25	3	1	120
8	19	8	25	3	2	145
8	19	8	25	3	3	150
8	19	8	25	4	1	160
8	19	8	25	4	2	155
8	19	8	25	4	3	175
8	19	8	25	5	1	110
8	19	8	25	5	2	148
8	19	8	25	5	3	150
8	19	8	25	6	1	110
8	19	8	25	6	2	94
8	19	8	25	6	3	90
8	19	9	34	1	1	120
8	19	9	34	1	2	90
8	19	9	34	1	3	125
8	19	9	34	2	1	110
8	19	9	34	2	2	95
8	19	9	34	2	3	100
8	19	9	34	3	1	95
8	19	9	34	3	2	85
8	19	9	34	3	3	110
8	19	9	34	4	1	130
8	19	9	34	4	2	80
8	19	9	34	4	3	120
8	19	9	34	5	1	100
8	19	9	34	5	2	95
8	19	9	34	5	3	95
8	19	9	34	6	1	85
8	19	9	34	6	2	80
8	19	9	34	6	3	60
8	19	10	41	1	1	130
8	19	10	41	1	2	105
8	19	10	41	1	3	110
8	19	10	41	2	1	115
8	19	10	41	2	2	95
8	19	10	41	2	3	90
8	19	10	41	3	1	85
8	19	10	41	3	2	85
8	19	10	41	3	3	90
8	19	10	41	4	1	125
8	19	10	41	4	2	75
8	19	10	41	4	3	110
8	19	10	41	5	1	95
8	19	10	41	5	2	90
8	19	10	41	5	3	85
8	19	10	41	6	1	85
8	19	10	41	6	2	85
8	19	10	41	6	3	65
8	19	11	29	1	1	125
8	19	11	29	1	2	95
8	19	11	29	1	3	125

8	19	11	29	2	1	110
8	19	11	29	2	2	90
8	19	11	29	2	3	95
8	19	11	29	3	1	80
8	19	11	29	3	2	80
8	19	11	29	3	3	90
8	19	11	29	4	1	120
8	19	11	29	4	2	50
8	19	11	29	4	3	105
8	19	11	29	5	1	75
8	19	11	29	5	2	80
8	19	11	29	5	3	80
8	19	11	29	6	1	70
8	19	11	29	6	2	75
8	19	11	29	6	3	60
8	19	12	36	1	1	115
8	19	12	36	1	2	95
8	19	12	36	1	3	100
8	19	12	36	2	1	85
8	19	12	36	2	2	80
8	19	12	36	2	3	75
8	19	12	36	3	1	70
8	19	12	36	3	2	65
8	19	12	36	3	3	75
8	19	12	36	4	1	70
8	19	12	36	4	2	55
8	19	12	36	4	3	80
8	19	12	36	5	1	50
8	19	12	36	5	2	80
8	19	12	36	5	3	90
8	19	12	36	6	1	70
8	19	12	36	6	2	70
8	19	12	36	6	3	60
8	19	14	18	1	1	70
8	19	14	18	1	2	85
8	19	14	18	1	3	95
8	19	14	18	2	1	100
8	19	14	18	2	2	90
8	19	14	18	2	3	90
8	19	14	18	3	1	75
8	19	14	18	3	2	70
8	19	14	18	3	3	80
8	19	14	18	4	1	80
8	19	14	18	4	2	70
8	19	14	18	4	3	75
8	19	14	18	5	1	20
8	19	14	18	5	2	85
8	19	14	18	5	3	70
8	19	14	18	6	1	60
8	19	14	18	6	2	75
8	19	14	18	6	3	60

8	19	14	55	1	1	125
8	19	14	55	1	2	90
8	19	14	55	1	3	100
8	19	14	55	2	1	80
8	19	14	55	2	2	90
8	19	14	55	2	3	80
8	19	14	55	3	1	80
8	19	14	55	3	2	80
8	19	14	55	3	3	60
8	19	14	55	4	1	105
8	19	14	55	4	2	50
8	19	14	55	4	3	90
8	19	14	55	5	1	80
8	19	14	55	5	2	115
8	19	14	55	5	3	65
8	19	14	55	6	1	-5
8	19	14	55	6	2	95
8	19	14	55	6	3	55
8	19	16	54	1	1	125
8	19	16	54	1	2	100
8	19	16	54	1	3	115
8	19	16	54	2	1	100
8	19	16	54	2	2	90
8	19	16	54	2	3	80
8	19	16	54	3	1	85
8	19	16	54	3	2	90
8	19	16	54	3	3	65
8	19	16	54	4	1	105
8	19	16	54	4	2	60
8	19	16	54	4	3	95
8	19	16	54	5	1	50
8	19	16	54	5	2	105
8	19	16	54	5	3	80
8	19	16	54	6	1	50
8	19	16	54	6	2	90
8	19	16	54	6	3	60
8	19	17	29	1	1	140
8	19	17	29	1	2	90
8	19	17	29	1	3	130
8	19	17	29	2	1	110
8	19	17	29	2	2	100
8	19	17	29	2	3	90
8	19	17	29	3	1	90
8	19	17	29	3	2	90
8	19	17	29	3	3	75
8	19	17	29	4	1	110
8	19	17	29	4	2	70
8	19	17	29	4	3	80
8	19	17	29	5	1	50
8	19	17	29	5	2	110
8	19	17	29	5	3	50

8	19	17	29	6	1	30
8	19	17	29	6	2	90
8	19	17	29	6	3	60
8	19	18	30	1	1	170
8	19	18	30	1	2	160
8	19	18	30	1	3	160
8	19	18	30	2	1	135
8	19	18	30	2	2	120
8	19	18	30	2	3	120
8	19	18	30	3	1	115
8	19	18	30	3	2	110
8	19	18	30	3	3	120
8	19	18	30	4	1	150
8	19	18	30	4	2	100
8	19	18	30	4	3	120
8	19	18	30	5	1	70
8	19	18	30	5	2	100
8	19	18	30	5	3	120
8	19	18	30	6	1	80
8	19	18	30	6	2	85
8	19	18	30	6	3	100
8	19	19	30	1	1	165
8	19	19	30	1	2	145
8	19	19	30	1	3	155
8	19	19	30	2	1	135
8	19	19	30	2	2	145
8	19	19	30	2	3	140
8	19	19	30	3	1	115
8	19	19	30	3	2	105
8	19	19	30	3	3	102
8	19	19	30	4	1	165
8	19	19	30	4	2	120
8	19	19	30	4	3	120
8	19	19	30	5	1	100
8	19	19	30	5	2	128
8	19	19	30	5	3	75
8	19	19	30	6	1	90
8	19	19	30	6	2	105
8	19	19	30	6	3	95
8	19	20	31	1	1	170
8	19	20	31	1	2	165
8	19	20	31	1	3	170
8	19	20	31	2	1	150
8	19	20	31	2	2	165
8	19	20	31	2	3	140
8	19	20	31	3	1	130
8	19	20	31	3	2	115
8	19	20	31	3	3	100
8	19	20	31	4	1	180
8	19	20	31	4	2	165
8	19	20	31	4	3	175

8	19	20	31	5	1	130
8	19	20	31	5	2	145
8	19	20	31	5	3	120
8	19	20	31	6	1	125
8	19	20	31	6	2	125
8	19	20	31	6	3	100
8	19	21	26	1	1	170
8	19	21	26	1	2	165
8	19	21	26	1	3	170
8	19	21	26	2	1	160
8	19	21	26	2	2	180
8	19	21	26	2	3	175
8	19	21	26	3	1	140
8	19	21	26	3	2	125
8	19	21	26	3	3	160
8	19	21	26	4	1	180
8	19	21	26	4	2	170
8	19	21	26	4	3	175
8	19	21	26	5	1	155
8	19	21	26	5	2	160
8	19	21	26	5	3	155
8	19	21	26	6	1	160
8	19	21	26	6	2	140
8	19	21	26	6	3	40
8	19	22	28	1	1	165
8	19	22	28	1	2	175
8	19	22	28	1	3	165
8	19	22	28	2	1	165
8	19	22	28	2	2	180
8	19	22	28	2	3	175
8	19	22	28	3	1	150
8	19	22	28	3	2	150
8	19	22	28	3	3	165
8	19	22	28	4	1	180
8	19	22	28	4	2	175
8	19	22	28	4	3	180
8	19	22	28	5	1	175
8	19	22	28	5	2	170
8	19	22	28	5	3	170
8	19	22	28	6	1	160
8	19	22	28	6	2	160
8	19	22	28	6	3	150
8	19	23	32	1	1	170
8	19	23	32	1	2	170
8	19	23	32	1	3	165
8	19	23	32	2	1	168
8	19	23	32	2	2	180
8	19	23	32	2	3	180
8	19	23	32	3	1	150
8	19	23	32	3	2	160
8	19	23	32	3	3	150

8	19	23	32	4	1	160
8	19	23	32	4	2	170
8	19	23	32	4	3	170
8	19	23	32	5	1	165
8	19	23	32	5	2	170
8	19	23	32	5	3	180
8	19	23	32	6	1	160
8	19	23	32	6	2	160
8	19	23	32	6	3	150
8	20	0	31	1	1	180
8	20	0	31	1	2	180
8	20	0	31	1	3	175
8	20	0	31	2	1	160
8	20	0	31	2	2	180
8	20	0	31	2	3	180
8	20	0	31	3	1	160
8	20	0	31	3	2	140
8	20	0	31	3	3	170
8	20	0	31	4	1	180
8	20	0	31	4	2	175
8	20	0	31	4	3	175
8	20	0	31	5	1	170
8	20	0	31	5	2	170
8	20	0	31	5	3	180
8	20	0	31	6	1	160
8	20	0	31	6	2	160
8	20	0	31	6	3	140
8	20	1	30	1	1	175
8	20	1	30	1	2	175
8	20	1	30	1	3	175
8	20	1	30	2	1	160
8	20	1	30	2	2	180
8	20	1	30	2	3	180
8	20	1	30	3	1	155
8	20	1	30	3	2	160
8	20	1	30	3	3	170
8	20	1	30	4	1	180
8	20	1	30	4	2	180
8	20	1	30	4	3	175
8	20	1	30	5	1	160
8	20	1	30	5	2	165
8	20	1	30	5	3	170
8	20	1	30	6	1	155
8	20	1	30	6	2	150
8	20	1	30	6	3	145
8	20	2	45	1	1	170
8	20	2	45	1	2	150
8	20	2	45	1	3	145
8	20	2	45	2	1	160
8	20	2	45	2	2	155
8	20	2	45	2	3	170

8	20	2	45	3	1	150
8	20	2	45	3	2	135
8	20	2	45	3	3	150
8	20	2	45	4	1	180
8	20	2	45	4	2	175
8	20	2	45	4	3	160
8	20	2	45	5	1	155
8	20	2	45	5	2	170
8	20	2	45	5	3	150
8	20	2	45	6	1	155
8	20	2	45	6	2	150
8	20	2	45	6	3	135
8	20	4	00	1	1	170
8	20	4	00	1	2	160
8	20	4	00	1	3	155
8	20	4	00	2	1	160
8	20	4	00	2	2	160
8	20	4	00	2	3	170
8	20	4	00	3	1	150
8	20	4	00	3	2	140
8	20	4	00	3	3	150
8	20	4	00	4	1	185
8	20	4	00	4	2	170
8	20	4	00	4	3	165
8	20	4	00	5	1	150
8	20	4	00	5	2	150
8	20	4	00	5	3	140
8	20	4	00	6	1	165
8	20	4	00	6	2	140
8	20	4	00	6	3	140
8	20	5	15	1	1	165
8	20	5	15	1	2	145
8	20	5	15	1	3	155
8	20	5	15	2	1	155
8	20	5	15	2	2	150
8	20	5	15	2	3	155
8	20	5	15	3	1	150
8	20	5	15	3	2	135
8	20	5	15	3	3	135
8	20	5	15	4	1	180
8	20	5	15	4	2	180
8	20	5	15	4	3	160
8	20	5	15	5	1	145
8	20	5	15	5	2	160
8	20	5	15	5	3	140
8	20	5	15	6	1	145
8	20	5	15	6	2	135
8	20	5	15	6	3	145
8	20	6	13	1	1	165
8	20	6	13	1	2	140
8	20	6	13	1	3	135

8	20	6	13	2	1	155
8	20	6	13	2	2	135
8	20	6	13	2	3	155
8	20	6	13	3	1	150
8	20	6	13	3	2	135
8	20	6	13	3	3	135
8	20	6	13	4	1	180
8	20	6	13	4	2	155
8	20	6	13	4	3	165
8	20	6	13	5	1	145
8	20	6	13	5	2	150
8	20	6	13	5	3	135
8	20	6	13	6	1	145
8	20	6	13	6	2	120
8	20	6	13	6	3	155
8	20	7	03	1	1	150
8	20	7	03	1	2	130
8	20	7	03	1	3	140
8	20	7	03	2	1	145
8	20	7	03	2	2	125
8	20	7	03	2	3	120
8	20	7	03	3	1	135
8	20	7	03	3	2	120
8	20	7	03	3	3	145
8	20	7	03	4	1	175
8	20	7	03	4	2	145
8	20	7	03	4	3	110
8	20	7	03	5	1	135
8	20	7	03	5	2	135
8	20	7	03	5	3	145
8	20	7	03	6	1	145
8	20	7	03	6	2	110
8	20	7	03	6	3	90
8	20	8	06	1	1	135
8	20	8	06	1	2	105
8	20	8	06	1	3	150
8	20	8	06	2	1	125
8	20	8	06	2	2	105
8	20	8	06	2	3	140
8	20	8	06	3	1	125
8	20	8	06	3	2	110
8	20	8	06	3	3	150
8	20	8	06	4	1	150
8	20	8	06	4	2	105
8	20	8	06	4	3	150
8	20	8	06	5	1	115
8	20	8	06	5	2	120
8	20	8	06	5	3	100
8	20	8	06	6	1	120
8	20	8	06	6	2	95
8	20	8	06	6	3	80

8	20	9	16	1	1	125
8	20	9	16	1	2	85
8	20	9	16	1	3	115
8	20	9	16	2	1	115
8	20	9	16	2	2	85
8	20	9	16	2	3	145
8	20	9	16	3	1	115
8	20	9	16	3	2	90
8	20	9	16	3	3	115
8	20	9	16	4	1	135
8	20	9	16	4	2	115
8	20	9	16	4	3	110
8	20	9	16	5	1	80
8	20	9	16	5	2	90
8	20	9	16	5	3	95
8	20	9	16	6	1	85
8	20	9	16	6	2	70
8	20	9	16	6	3	70
8	20	10	01	1	1	125
8	20	10	01	1	2	90
8	20	10	01	1	3	110
8	20	10	01	2	1	120
8	20	10	01	2	2	80
8	20	10	01	2	3	135
8	20	10	01	3	1	115
8	20	10	01	3	2	90
8	20	10	01	3	3	110
8	20	10	01	4	1	135
8	20	10	01	4	2	85
8	20	10	01	4	3	110
8	20	10	01	5	1	90
8	20	10	01	5	2	90
8	20	10	01	5	3	95
8	20	10	01	6	1	85
8	20	10	01	6	2	85
8	20	10	01	6	3	55
8	20	11	45	1	1	130
8	20	11	45	1	2	95
8	20	11	45	1	3	90
8	20	11	45	2	1	105
8	20	11	45	2	2	85
8	20	11	45	2	3	100
8	20	11	45	3	1	100
8	20	11	45	3	2	90
8	20	11	45	3	3	95
8	20	11	45	4	1	135
8	20	11	45	4	2	70
8	20	11	45	4	3	110
8	20	11	45	5	1	115
8	20	11	45	5	2	100
8	20	11	45	5	3	105

8	20	11	45	6	1	60
8	20	11	45	6	2	90
8	20	11	45	6	3	90
8	20	12	55	1	1	125
8	20	12	55	1	2	110
8	20	12	55	1	3	110
8	20	12	55	2	1	105
8	20	12	55	2	2	110
8	20	12	55	2	3	85
8	20	12	55	3	1	80
8	20	12	55	3	2	95
8	20	12	55	3	3	80
8	20	12	55	4	1	125
8	20	12	55	4	2	80
8	20	12	55	4	3	102
8	20	12	55	5	1	115
8	20	12	55	5	2	100
8	20	12	55	5	3	105
8	20	12	55	6	1	50
8	20	12	55	6	2	80
8	20	12	55	6	3	75
8	20	14	38	1	1	180
8	20	14	38	1	2	100
8	20	14	38	1	3	105
8	20	14	38	2	1	180
8	20	14	38	2	2	85
8	20	14	38	2	3	90
8	20	14	38	3	1	90
8	20	14	38	3	2	85
8	20	14	38	3	3	95
8	20	14	38	4	1	120
8	20	14	38	4	2	75
8	20	14	38	4	3	100
8	20	14	38	5	1	105
8	20	14	38	5	2	110
8	20	14	38	5	3	80
8	20	14	38	6	1	140
8	20	14	38	6	2	85
8	20	14	38	6	3	65
8	20	15	40	1	1	130
8	20	15	40	1	2	100
8	20	15	40	1	3	100
8	20	15	40	2	1	100
8	20	15	40	2	2	85
8	20	15	40	2	3	80
8	20	15	40	3	1	85
8	20	15	40	3	2	85
8	20	15	40	3	3	65
8	20	15	40	4	1	50
8	20	15	40	4	2	75
8	20	15	40	4	3	100

8	20	15	40	5	1	95
8	20	15	40	5	2	90
8	20	15	40	5	3	90
8	20	15	40	6	1	45
8	20	15	40	6	2	70
8	20	15	40	6	3	60
8	20	16	24	1	1	130
8	20	16	24	1	2	100
8	20	16	24	1	3	100
8	20	16	24	2	1	90
8	20	16	24	2	2	85
8	20	16	24	2	3	75
8	20	16	24	3	1	85
8	20	16	24	3	2	85
8	20	16	24	3	3	75
8	20	16	24	4	1	125
8	20	16	24	4	2	80
8	20	16	24	4	3	95
8	20	16	24	5	1	105
8	20	16	24	5	2	180
8	20	16	24	5	3	175
8	20	16	24	6	1	50
8	20	16	24	6	2	95
8	20	16	24	6	3	60
8	20	19	30	1	1	150
8	20	19	30	1	2	135
8	20	19	30	1	3	120
8	20	19	30	2	1	150
8	20	19	30	2	2	135
8	20	19	30	2	3	125
8	20	19	30	3	1	140
8	20	19	30	3	2	140
8	20	19	30	3	3	75
8	20	19	30	4	1	130
8	20	19	30	4	2	85
8	20	19	30	4	3	120
8	20	19	30	5	1	70
8	20	19	30	5	2	135
8	20	19	30	5	3	90
8	20	19	30	6	1	85
8	20	19	30	6	2	110
8	20	19	30	6	3	70
8	20	20	30	1	1	160
8	20	20	30	1	2	150
8	20	20	30	1	3	145
8	20	20	30	2	1	155
8	20	20	30	2	2	150
8	20	20	30	2	3	135
8	20	20	30	3	1	140
8	20	20	30	3	2	125
8	20	20	30	3	3	165

8	20	20	30	4	1	170
8	20	20	30	4	2	135
8	20	20	30	4	3	180
8	20	20	30	5	1	150
8	20	20	30	5	2	155
8	20	20	30	5	3	140
8	20	20	30	6	1	120
8	20	20	30	6	2	150
8	20	20	30	6	3	115
8	20	20	45	1	1	170
8	20	20	45	1	2	130
8	20	20	45	1	3	140
8	20	20	45	2	1	165
8	20	20	45	2	2	140
8	20	20	45	2	3	120
8	20	20	45	3	1	135
8	20	20	45	3	2	125
8	20	20	45	3	3	135
8	20	20	45	4	1	85
8	20	20	45	4	2	145
8	20	20	45	4	3	160
8	20	20	45	5	1	160
8	20	20	45	5	2	155
8	20	20	45	5	3	135
8	20	20	45	6	1	120
8	20	20	45	6	2	150
8	20	20	45	6	3	160
8	21	14	45	1	1	140
8	21	14	45	1	2	110
8	21	14	45	1	3	110
8	21	14	45	2	1	85
8	21	14	45	2	2	95
8	21	14	45	2	3	90
8	21	14	45	3	1	65
8	21	14	45	3	2	85
8	21	14	45	3	3	95
8	21	14	45	4	1	120
8	21	14	45	4	2	80
8	21	14	45	4	3	95
8	21	14	45	5	1	75
8	21	14	45	5	2	70
8	21	14	45	5	3	75
8	21	14	45	6	1	35
8	21	14	45	6	2	80
8	21	14	45	6	3	80
8	21	15	16	1	1	140
8	21	15	16	1	2	100
8	21	15	16	1	3	180
8	21	15	16	2	1	85
8	21	15	16	2	2	95
8	21	15	16	2	3	110

8	21	15	16	3	1	75
8	21	15	16	3	2	90
8	21	15	16	3	3	85
8	21	15	16	4	1	130
8	21	15	16	4	2	90
8	21	15	16	4	3	95
8	21	15	16	5	1	55
8	21	15	16	5	2	120
8	21	15	16	5	3	65
8	21	15	16	6	1	10
8	21	15	16	6	2	72
8	21	15	16	6	3	70

APPENDIX B

A PAPER SUBMITTED

TO

IEEE TRANSACTION ON GEOSCIENCES AND REMOTE SENSING

POLARIZATION UTILIZATION IN THE
MICROWAVE INVERSION OF LEAF
ANGLE DISTRIBUTIONS

by

N. S. Chauhan and R. H. Lang

Department of Electrical Engineering and Computer Science
The George Washington University
Washington, DC 20052

ABSTRACT

The inverse problem of deducing the inclination angle distribution of leafy vegetation has been investigated using L band multipolarization backscattered model data. The modeling procedure replaces canopy leaves with thin circular dielectric discs. The Born approximation is then employed to establish a linear relationship between the radar backscattering coefficients and the leaf inclination angle distribution. Due to the ill-posedness of the problem, the Phillips-Twomey regularization method with second order smoothing condition is used. The inversion of the leaf angle distribution is carried out for horizontal, vertical and cross-polarizated model data. It is shown that results of the inversion using vertical and cross polarized model data are comparable to the horizontal inversion results obtained in a previous paper. All these single polarization inversions require data at approximately 10 incident angles. Since radar observations from space are usually limited to 3 or 4 incidence angles, a method combining the three polarization inversions has been devised which reduces the number of incidence angles required by a factor of three.

1. INTRODUCTION

Vegetation canopies such as soybeans have been modeled effectively in the microwave frequency regime by replacing their leaves with dielectric discs [1-4]. The relationship between backscattered energy and disc parameters have been found by using the Foldy-Lax and the distorted Born approximation. For the L-band region of the spectrum, the skin depth of the vegetation such as soybeans is large. Consequently, the distorted Born equations reduce to those of the Born between the backscattering coefficient and the inclination angle-area probability density distribution can be established. The ill-posed nature of the problem requires the application of some regularization technique. The Phillips-Twomey [5-7] regularization technique with second order difference smoothing condition is used here. This technique has been successfully employed by Lang and Saleh[8] who treated the case where horizontally polarized data is used to determine the eaf angle distribution. Other researchers[9-10] at optical frequencies have employed similar techniques to determine properties of the atmosphere's constitutents. In addition, at optical frequencies the remote determination of canopy leaf inclination angle and leaf area index data has been studied by Goel & Thompson [11]. His methods, however, differ from those presented here.

This paper has been motivated, in part, by the need to apply the inversion method used in [8] to vertical and cross polarized data. Comparison of the inclination angle distributions obtained for these polarized data sets along with the results of the horizontal case obtained in [8] will give a indication of which polarization type is the most sensitive. A second motivation for the paper is to use the polarized model data sets collectively in order to reduce the number of incident angles at which data is required. Presently, shuttle data can only be obtained at three or four incident angles. However, for reasonable accuracy approximately 10 incident angles are required to estimate the leaf angle distribution over the range of inclination angles from 0° to 90°. By utilizing the polarization data collectively one obtains three times as much data at each angle of incidence and thus the number of incidence angles can be reduced by a factor of three. Finally, it should be mentioned that although the present paper is devoted to determining the leaf inclination angle distribution for a known leaf area distribution, the method, as presented in [8], can also be used to find the leaf radius or area distribution for a known leaf inclination angle distribution. In addition, it has been shown in [12], that if data is given over incidence angle and frequency, the joint leaf inclination angle and radius distribution can be determined.

The paper is divided into six sections. The direct backscattering problem for horizontal, vertical and cross polarizations is given in section 2. In section 3 the inversion problem is formulated and in section 4 the continuous formulation is discretizated. In this section the Phillips-Twomey inversion solution for the leaf inclination angle distribution is given in terms of a regularization parameter and an algorithm(residue method) is presented for determining the "best" regularization parameter. Finally, in section 5 the numerical results for single and combined polarization inversions are discussed and compared.

2. BACKSCATTERING MODEL

The vegetation layer is modeled by a slab of lossy circular dielectric discs having random positions and orientation (Fig. 1). The discs are assumed to be independent of one another and have prescribed orientation statistics. Stems are not considered in the present model and it is assumed they can be neglected. The ground under the vegetation is taken to be a flat lossy dielectric half space. Rough surface effects are considered to be negligible. This is the case for many mature soybean canopies in the L band region of the spectrum. The direct problem for the slab of random discs consists of computing the backscattering coefficients by using the Born approximation for the horizontal, vertical and cross-polarized cases.

The radar backscattering coefficients for a canopy of discrete scatterers have been obtained by the distorted Born approximation[8,13,14]. For the L-band region of the spectrum, the skin depth for most agricultural crops is large. Under this assumption, the backscattering coefficients simplify to the Born theory expressions which are given by:

$$\sigma^{o}_{pq} = \sigma^{o}_{pqd} + \sigma^{o}_{pqr} + \sigma^{o}_{pqdr} \tag{1}$$

where

$$\sigma^{o}_{pqd} = \rho d\sigma_{pqd} \tag{2}$$

$$\sigma^{o}_{pqr} = \rho d\sigma_{pqr} |R^{(p)}_{s}|^{2} |R^{(q)}_{s}|^{2} \tag{3}$$

$$\sigma^{o}_{pqdr} = \rho d\Big(\sigma_{pqdr1}|R_{g}^{(p)}|^2 + \sigma_{pqdr2}|R_{g}^{(q)}|^2 + 2\mathrm{Re}(\sigma_{pqdr3}R_{g}^{(p)*}R_{g}^{(q)})\Big) \tag{4}$$

$$p,q \in \{h,v\}$$

In the above equations σ^{o}_{pq}, σ_{pq} are the backscattering coefficients for the slab and the scattering cross sections for the discs respectively. The suffixes pqd, pqdr are for direct, reflected and direct-reflected coefficients respectively for a wave of like (p=q) or unlike (p≠q) polarization. The density of scatterers in the layer is ρ, the thickness of the canopy is d and $R_{g}^{(p)}$ is the Fresnel reflection coefficient from the ground interface for a wave of polarization p.

The scattering cross sections σ_{pq} appearing in (2)-(4) are for a single dielectric disc which is averaged over size and orientation. The cross sections can be expressed directly in terms of the disc's dyadic scattering amplitude $\underline{f}(\hat{o},\hat{i})$. Here $\hat{i}$ and $\hat{o}$ are unit vectors in the directions of the incident and scattered waves. The direct and reflected backscattering cross sections are given by:

$$\sigma_{pqd} = 4\pi\overline{|\hat{p}^{-}\cdot\underline{f}(-\hat{i}^{-},\hat{i}^{-})\cdot\hat{q}^{-}|^2} \tag{5}$$

and

$$\sigma_{pqr} = 4\pi\overline{|\hat{p}^{+}\cdot\underline{f}(-\hat{i}^{+},\hat{i}^{+})\cdot\hat{q}^{+}|^2} \tag{6}$$

The direct-reflected or interference terms are bistatic in nature and are given by:

$$\sigma_{pqdr1} = 4\pi\overline{|\hat{p}^{+}\cdot\underline{f}(-\hat{i}^{+},\hat{i}^{-})\cdot\hat{q}^{-}|^2} \tag{7a}$$

$$\sigma_{pqdr2} = 4\pi\overline{|\hat{p}^{-}\cdot\underline{f}(-\hat{i}^{-},\hat{i}^{+})\cdot\hat{q}^{+}|^2} \tag{7b}$$

and

$$\sigma_{pqdr3} = 4\pi\overline{\left[\hat{p}^{+}\cdot\underline{f}(-\hat{i}^{+},\hat{i}^{-})\cdot\hat{q}^{-}\right]\left[\hat{p}^{-}\cdot\underline{f}^{*}(-\hat{i}^{-},\hat{i}^{+})\cdot\hat{q}^{+}\right]} \tag{7c}$$

The unit vectors $\hat{i}^{\pm}$ used in the above formulas are defined in Fig. 1. In this figure it is seen that $\hat{i}^{-}$ is the direction of the incident wave on the canopy while $\hat{i}^{+}$ is in the direction of the incident wave's specular reflection from the ground interface. Thus each disc has two separate waves incident upon it. The directions $-\hat{i}^-$ and $-\hat{i}^+$ are the directions in which waves scattered by the discs return to the radar. The scattered wave either returns to the radar directly in the case of $-\hat{i}^-$ or is specularly reflected by the interface before returning as is the case with $-\hat{i}^+$. The polarization vectors $\hat{h}^{\pm}$ and $\hat{v}^{\pm}$ are also shown in Fig. 1; they are used in place of $\hat{p}^{\pm}$ and $\hat{q}^{\pm}$ in the formulas shown above. Finally, the super bar over eqs.(5-7) represents the average over the disc radius and orientation angles.

3. FORMULATION OF INVERSE PROBLEM

An examination of equations (1)-(7) shows that a linear integral relationship exists between the backscattering coefficient, σ^{o}_{pq}, and the joint probability density of disc radius and inclination angle. Symbolically, it is expressed as

$$\sigma^{o}_{pq}(\theta_{o}) = \int K_{pq}(\theta_{o}, A, \theta) p(A, \theta) dA d\theta \tag{8}$$

where K_{pq} is the kernel function and $p(A,\theta)$ is the joint probability density function. Here A is the disc radius and θ is the disc inclination angle. This angle is defined by the z axis and the normal to the disc. Implicit in the integral relationship is the assumed independence of the disc's azimuth orientation angle ϕ from θ and A(see [8] for more details).

The kernel function is expressed as a sum of direct, reflected and direct-reflected parts:

$$K_{pq} = K_{pqd} + K_{pqr} + K_{pqdr} \tag{9}$$

Each term in the sum corresponds to the appropriate scattering coefficient component as given in (2)-(4). Explicit expressions for the kernel components can be found by substituting the scattering amplitude for a thin disc, as given in [8],into the cross-section formulas in equations (5)-(7)(see [15] for corrections to [8]). The final expressions result when these cross-sections are used in equations (2)-(4).For the direct component the kernel function is given by:

$$K_{pqd} = \int_0^{2\pi} \frac{d\phi}{2\pi} Q_{pqd} \bar{S}^2(\nu_d) \tag{10}$$

where

$$Q_{hhd} = \beta |1 - \zeta q_1^2|^2 \tag{11a}$$

$$Q_{vvd} = \beta |1 - \zeta (q_2 - q_3)^2|^2 \tag{11b}$$

$$Q_{vhd} = \beta |\zeta|^2 q_1^2 (q_2^2 + q_3^2) \tag{11c}$$

with

$$q_1 = \sin\theta\cos\phi$$
$$q_2 = \sin\theta\cos\theta_0\sin\phi \qquad \text{and} \qquad \zeta = \frac{\chi}{\chi + 1} \tag{12}$$
$$q_3 = \cos\theta\sin\theta_0$$

Here $\bar{S}^2(\nu_d)$ is the pattern function for a circular disc evaluated in the backscatterd direction(see,[8]), χ is the susceptibility of the disc dielectric material and the constant β is given by

$$\beta = \frac{\rho d k_0^4 |\chi|^2 T^2}{4\pi} \tag{13}$$

where k_0 is the free space wavenumber and T is the disc thickness. It should also be noted that the discs have been assumed to be uniformly distributed in the azimuthal coordinate.

The reflected component of the kernel can be simply written in terms of the direct component as follows:

$$K_{pqr} = |R_g^{(p)}|^2 |R_g^{(q)}|^2 K_{pqd} \tag{14}$$

The simple relationship between the reflected and direct component is the result of the assumed uniformity in ϕ.

The direct-reflected componennt of the kernel function is given below separately for like and cross polarized cases. For like polarization (p = q) the kernel is:

$$K_{ppdr} = 4 |R_g^{(p)}|^2 \int_0^{2\pi} \frac{d\phi}{2\pi} Q_{ppdr} \bar{S}^2(\nu_{bi}) \tag{15}$$

where

$$Q_{hhdr} = Q_{hhd} \tag{16a}$$

$$Q_{vvdr} = \beta|q_4 + \zeta(q_2 - q_3)|^2 \tag{16b}$$

with

$$q_4 = \sin^2\theta_i - \cos^2\theta_i \tag{17}$$

The expression for the kernel in the cross polarized case($p \neq q$) is:

$$K_{pqdr} = \int_0^{2\pi}\Big[|R_s^{(p)}|^2 Q^+_{pqdr} + |R_s^{(q)}|^2 Q^+_{pqdr} + 2\mathrm{Re}(R_s^{(p)*}R_s^{(q)})Q^-_{pqdr}\Big]\,\bar{S}^2(\nu_{bi}) \tag{18}$$

where

$$Q^{\pm}_{pqdr} = \beta|\zeta|^2 q_1(q_3^2 \pm q_2^2) \tag{19}$$

In (15) and (18) the pattern function $\bar{S}^2(\nu_{bi})$ is evaluated in the bistatic direction(see, [8]).

The integral equation given in (8) relates the backscattering coefficient to the joint distribution over disc radius and inclination angle. Since the radius distribution will be assumed as known apriori, (8) can be rewritten as:

$$\sigma^{o}_{pq}(\theta_o) = \int K^{(\theta)}_{pq}(\theta_o,\theta)p(\theta)d\theta \tag{20}$$

where

$$K^{(\theta)}_{pq}(\theta_o,\theta) = \int K_{pq}(\theta_o,A,\theta)p(A|\theta)dA \tag{21}$$

Here $p(A|\theta)$ is the conditional probability of the radius given that the inclination angle is known. If A and θ are independent than $p(A|\theta)$ reduces to $p(A)$.

Equation (20) represents the direct problem. Since it will be assumed that information is available to determine the kernel $K^{(\theta)}_{pq}$ completely, the inverse problem involves the determination of the probability density function of disc inclination given the backscattering cross-section at different angles of incidence. This paper deals with the determination and comparison of $p(\theta)$ for horizontal, vertical, cross polarization cases. The basic question to be answered is: Given the backscattering coefficient at just a few angles of incidence, can we utilize the information from different polarizations to find the orientation distribution of the discs with reasonable accuracy?

4. INVERSION TECHNIQUE

The inversion of Eq.(20) for $p(\theta)$ is a two part process. First the integral equation must be discretized and then a stable inverse for the discrete system must be constructed. The discretization process is necessary since backscattering data is available at only a finite number of points. To invert the integral equation exactly, data is needed at all backscattered angles. Since this is not possible, the integral equation is replaced by a finite system of algebraic equations.

Before discretizing $p(\theta)$ it is necessary to discuss the range of values that θ can assume. For a general leaf shape θ would normally take on values between 0 and π. For the special case of circular discs considered here, the normal to the leaf can always be chosen so that θ is less than $\pi/2$. Thus, in this paper it will be assumed that $p(\theta) = 0$ for $\pi/2 < \theta \leq \pi$.

The first step in the discretization process is to approximate the unknown function $p(\theta)$ by a step function $\hat{p}(\theta)$ having N levels

$$\hat{p}(\theta) = \sum_{n=1}^{N} p_n \, \mathrm{rect}_n(\theta) \tag{22}$$

where

$$\mathrm{rect}_n(\theta) = \begin{cases} 1\,, & (n-1)\Delta\theta < \theta < n\Delta\theta \\ 0\,, & \text{elsewhere} \end{cases} \tag{23}$$

with $\Delta\theta = \pi/2N$. The step heights p_n are chosen so that they equal the probability density function at the center point of the interval, i.e. $p_n = p(\theta_n)$ where $\theta_n = (n-1/2)\Delta\theta$. A plot of a typical $p(\theta)$ and $\hat{p}(\theta)$ is shown in Fig.2. The abscissa is plotted in degrees rather than radians while in the text θ and $\Delta\theta$ are in radians. Thus the plotted density functions must be multipled by a normalization factor of $\pi/180$. This scaling factor has not been used in Figs. 4, 5 & 8 of [8]. Thus these figures are incorrect as they appear and must have their ordinate values multiplied by $\pi/180$. This insures that the area under the density function is one.

The next step in the discretization process is to replace the continuous density $p(\theta)$ which appears in (21) by its discrete estimate $\hat{p}(\theta)$. Proceeding, (21) becomes

$$\hat{\sigma}_{pq}(\theta_o) = \int_0^{\pi/2} K_{pq}^{(\theta)}(\theta_o,\theta)\,\hat{p}(\theta)d\theta \tag{24}$$

where $\hat{\sigma}_{pq}$ is the discrete estimated value of σ^o_{pq}. Now using (22) in (24), one finds

$$\hat{\sigma}_{pq}(\theta_o) = \sum_{n=1}^{N} K_n^{pq}(\theta_o)p_n \tag{25}$$

where

$$K_n^{pq}(\theta_o) = \int_{\theta_n^-}^{\theta_n^+} K_{pq}^{(\theta)}(\theta_o,\theta)d\theta \tag{26}$$

with $\theta_n^{\pm} = \theta_n \pm \Delta\theta/2$. It should also be noted that $K_n^{pq}(\theta_0)$ is a continuous function of θ_0 that can take on values between 0 and $\pi/2$.

If the backscattering coefficients for a particular polarization type pq are measured at M incident angles and (25) is evaluated at each of these angles then a system of M linear equations with N unknowns ($M \geq N$) is generated. In matrix form this system is written as

$$
\begin{bmatrix} \hat{\sigma}_{pq}(\theta_{o1}) \\ \vdots \\ \hat{\sigma}_{pq}(\theta_{oM}) \end{bmatrix}
=
\begin{bmatrix} \text{M x N kernel} \\ \text{matrix of} \\ \text{pq coefficients} \end{bmatrix}
\begin{bmatrix} p_1 \\ \vdots \\ p_N \end{bmatrix}
\tag{27}
$$

where the M×N matrix coefficients are $K_n^{pq}(\theta_{0m})$. It is seen that there is a different system of equations for each polarization type. Thus by using only one polarization type it should be possible to determine the inclination angle probability density function (pdf). This pdf can be obtained independently for each polarization data type. For backscattering coefficients sampled at L incident angles, a system of M = 3L equations can be generated. The matrix form of this mixed polarization system is:

$$
\begin{bmatrix} \hat{\sigma}_{hh}(\theta_{o1}) \\ \vdots \\ \hat{\sigma}_{hh}(\theta_{oL}) \\ \hline \hat{\sigma}_{vv}(\theta_{o1}) \\ \vdots \\ \hat{\sigma}_{vv}(\theta_{oL}) \\ \hline \hat{\sigma}_{hv}(\theta_{o1}) \\ \vdots \\ \hat{\sigma}_{hv}(\theta_{oL}) \end{bmatrix}
=
\begin{bmatrix} \text{L x N kernel} \\ \text{matrix of} \\ \text{hh coefficients} \\ \hline \text{L x N kernel} \\ \text{matrix of} \\ \text{vv coefficients} \\ \hline \text{L x N kernel} \\ \text{matrix of} \\ \text{hv coefficients} \end{bmatrix}
\begin{bmatrix} p_1 \\ \vdots \\ p_N \end{bmatrix}
\tag{28}
$$

This later approach demonstrates that to find an inversion with multi-polarization data, only one third the number of incident angles will be required compared to using one polarization type alone. Symbolically either (27) or (28) can be represented by the general system

$$\hat{\sigma} = K \hat{p} \tag{29}$$

where $\hat{\sigma}$ and $\hat{p}$ are M x 1 and N x 1 columns vectors respectively and K is a M x N rectangle matrix.

In practical situations, the backscattering coefficient data is corrupted by measurement noise. In the present study the effect of this measurement noise is simulated by adding random fluctuations to each component of $\hat{\sigma}$. To be specific, if the noisy backscattering data vector is denoted by $\hat{\sigma}_\delta$ then each component of this vector is given by

$$\hat{\sigma}_{\delta m} = \hat{\sigma}_m (1 + \delta \eta_m) \tag{30}$$

where $\hat{\sigma}_m$ and $\hat{\sigma}_{\delta m}$ are the components of the row vectors $\hat{\sigma}$ and $\hat{\sigma}_\delta$, respectively, the η_m, n = 1,..., M, are zero mean independent random variables distributed uniformily between -1 and 1 and δ is a constant factor proportional to the percentage of noise added to each component of the backscattering coefficient vector $\hat{\sigma}$. Thus simulated values of the measured backscattering coefficient are generated by first using the model equation (29) with an assumed probability vector $\hat{p}$ to calculate $\hat{\sigma}$. The noise is then added via (30).

For a given level of noise δ the solution to (29) with $\hat{\sigma}$ replaced by $\hat{\sigma}_\delta$ normally provides a better approximation to the probability density as the system size N grows larger. Since (29) is derived from a Fredholm integral equation of the first kind, this system becomes ill-conditioned as N increases. As a result the inverted density may not tend toward the true or assumed density function as N increases. To obtain a meaningful solution to such an ill-conditioned system, the kernal K needs to be regularized. In the present study, the Phillips-Twomey technique [5,6] has been used for the regularization of K. The smoothing of K is achieved through a second order smoothing matrix H [7]. The amount of smoothing is controlled by a regularization parameter α whose determination is discussed below. Following the formulation given by Twomey [7] the regularized solution is

$$\hat{p}_\alpha = (K^T K + \alpha H)^{-1} K^T \hat{\sigma}_\delta \tag{31}$$

where the superscript T stands for transpose of the matrix.

Equation (31) is used to compute the estimated probability vector, $\hat{p}_\alpha$, for a monotonic sequence of α values. This vector is then used to calculate the estimated data vector, $\hat{\sigma}_\alpha = K\hat{p}_\alpha$, for each α. The best α is then chosen such that the solution has the least amount of smoothing (smallest α) with the constraint that the residual, $\|\hat{\sigma}_\alpha - \hat{\sigma}_\delta\|$, remain below some prescribed value related to the noise level δ. This method for determining α is known as the residue method and was first used by Morozov[16] and later discussed by Miller[17].

The average difference between the assumed density, $p(\theta)$, and the inverted density, $\hat{p}_\alpha(\theta)$, is used as a overall measure of the goodness of the inversion process. The square root of this average is called the total error, ϵ_t. It is defined as

$$\epsilon_t = \|p(\theta) - \hat{p}_\alpha(\theta)\| \tag{32}$$

where the vertical bars indicate the L_2 norm over the interval $[0, \pi/2]$. More specifically, we have

$$\epsilon_t = \sqrt{\int_0^{\pi/2} [p(\theta) - \hat{p}_\alpha(\theta)]^2 d\theta} \tag{33}$$

The function $\hat{p}_\alpha(\theta)$, used in (32) and (33), is obtained by putting the components of $\hat{p}_\alpha$ in (22), i.e.,

$$\hat{p}_\alpha(\theta) = \sum_{n=1}^{N} \hat{p}_{\alpha n} \, \mathrm{rect}_n(\theta) \tag{34}$$

The total error can be viewed as being composed of two distinct parts. First there is the error due to the quadrature approximation [see, (22)]. It is denoted by ϵ_q and is defined as the L_2 norm of the difference between $\hat{p}(\theta)$ and $p(\theta)$, i.e.,

$$\epsilon_q = \|\hat{p}(\theta) - p(\theta)\| \tag{35}$$

The second error is made as a result of the measurement noise and the ill-conditioned nature of the system of equations. This regularization error is denoted by ϵ_α and is the L_2 norm of the difference between the quadrature approximation and the inverted density function, i.e.,

$$\epsilon_{\alpha} = \|\hat{p}_{\alpha}(\theta) - \hat{p}(\theta)\| \tag{36}$$

It should be noted that this error is a function of the regularization parameter as well as the percentage of noise δ.

The interrelationship between these errors can be better understood by employing the triangular inequality as follows:

$$\|p_{\alpha}(\theta) - \hat{p}(\theta)\| = \|p_{\alpha}(\theta) - \hat{p}(\theta) + \hat{p}(\theta) - p(\theta)\|$$

$$\leq \|p_{\alpha}(\theta) - \hat{p}(\theta)\| + \|\hat{p}(\theta) - p(\theta)\| \tag{37}$$

or

$$\epsilon_t \leq \epsilon_q + \epsilon_{\alpha} \tag{38}$$

Thus it is seen that the sum of the quadrature and regularization errors provide an upper bound for the total error. It will be seen in the next section that this bound is quite tight in cases of practical interest.

5. RESULTS AND DISCUSSION

The model parameters in this problem are chosen to represent a mature soybean crop. The dielectric constant $\epsilon_r = 40.8 + i3.26$ at a frequency of 1.5 GHz with a water volume filling factor of 0.7 has been calculated from DeLoor formula [see, 18]. The discs have a radius of 4 cm and thickness 0.2 mm. These discs having a density of 1000 m^{-3} are assumed to be placed in a slab of 0.6 m thick. Since the discs have been assumed to be of only one size, the probability distribution of radii can be written as $p(A) = \delta(A-A_o)$, where A_o represents average radius the discs. It is also assumed that the leaves are uniformily distributed in the azimuthal orientation angle. Smith[19] has given several inclination angle distribution for plants. Since the present study is aimed at the demonstration of an inversion method, we have assumed the following simple probability distribution for the inclination of the leaves

$$p(\theta = \begin{cases} \cos\theta \ , & 0 \le \theta < \pi/2 \\ 0 \ , & \pi/2 \ge \theta < \pi \end{cases} \tag{39}$$

which is representative of those found in nature. The smooth curves in Figs. 2,4 and 5 represent Eq. (39).

A noise value of $\delta = .05$ or 5 percent has been chosen to corrupt the model values of the backscattering coefficients. The inversion of this simulated noisy backscattered data is examined in two cases. First, the probability density function is found for each polarization separately. The assumption is implicitly made that data from only one polarization is available. The results are compared. Second, a comparison between the inversion obtained with three and with nine point backscattered data of one polarization is made. It is then shown that an inversion performed with three point multipolarization data is more accurate than an inversion using only three point single polarization data.

CASE 1

The backscattering coefficients as a function of the angle of incidence are shown in Fig. 3(a,b,c) for horizontal, vertical and cross polarization respectively. The continuous line represents the backscattered data obtained by using the integral equation, (20), while the circles represent the data obtained with the discretized quadrature approximation, (25). The discrete data is sampled at nine angles of incidence. It can be noticed from Fig. 3 that for all three polarizations, the values calculated by the Born approximation and its quadrature approximation are close to each other.

The backscatter data is corrupted with 5 percent noise and is inverted to find probability density with second order difference constraints for various regularization parameters. The plots of probability density vs. inclination angle of the leaves for the best value of α are shown in Fig. 4. The procedure is repeated for all the three polarization cases. It is seen that for all the polarizations, the inverted probability density approximates the continuous (true) probability density quite well. A quantitative estimate, shown in Table 1 (top three lines), sums up the various error estimates corresponding to the best value of regularization parameter. The quadrature error, ϵ_q, is a constant quantity and does not depend upon regularization parameter or the type of polarization. It only depends upon the number of sampling intervals of the probability density which is nine for the top three lines of the Table 1. By choosing a bette interpolation technique for discretization of $p(\theta)$ one can further reduce this error. The other component of the error, ϵ_α, is better for horizontal and vertical polarization than for the cross polarized case showing that like polarized systems appear to do better than cross polarized systems. This observation, however is based on these very limited results.

To see how good is the present inversion technique vis-a-vis noise, we have carried out the inversion process for noise level of 25 and 50 percent for horizontal polarization. It is found that as noise level increases, the inverted probability density departs more and more from its true value. But even for a noise of 50%, the agreement does not look bad (Fig.4(a)). In a practical situation, the distribution of noise may not be uniform at all incident angles as assumed here for simulation of backscattered data and thus the results might look different than those shown.

CASE 2

For an actual satellite observation, the number of incident angles is not large. For example, in the SIR-B experiment, where the antenna was tilted mechanically to acquire images at selected incidence angles, the number of angles was just four [20]. Bearing this in mind, we have attempted the inverse problem for just three incident angles.

Using the quadrature approximation in (25), the true backscattered data is discretized for horizontally polarized waves having incident angles of 15°, 45°, 75°. For the inverse problem, the probability density is discretized at three angles of inclination. The smoothing matrix H which arises due to the second order smoothing condition will now be a 3x3 matrix [7]. The regularized solution [Eq. (31)], is obtained for 5 percent noise and the results are shown in Fig. 5(a). The total error, ϵ_t between the continuous (true) and the inverted probability density is found to be equal to 0.023. Comparing this error from three-point data to that of nine point data for the horizontal polarization reveals that the results in the three-point case are less accurate by a factor of five (see Table 1). In other words, the data sampled at three points does not give as accurate value of the inverted parameter. As would be suspected an examination of Table 1 shows that most of the incresed error lies in the quadrature component.

One solution to this problem is to increase the number of data points. Practically, this may not be feasible beyond a particular limit. We have attempted the problem through the utilization of data sampled at three polarizations for three incident angles. The probability density is discretized at nine inclination angles. The kernel for the combined polarization is set up for nine angles of inclination and three angles (for each polarization) of incidence (eq. (28) with L=3 & N=9). The backscattering coefficient for the combined polarization corrupted with 5 percent noise is inverted to find the probability density distribution of the leaves for the combined polarization. The resulting probability density is shown in Fig. 5(b). The error between continuous (true) and inverted probability density now is $\epsilon_t = 0.0023$. Comparing this with the three-point case with only one polarization (Table 1) it is found that combined polarization gives almost 10 times more accurate results than the single polarization with the same number of observation angles. The quadrature error ϵ_q which had increased in single polarization case gets reduced again in three polarization case.

The present study showed that polarization utilization greatly improved the estimation of the inverted probability density when the observational data was limited to fewer incident angles. It is suspected that results could be further improved if there was a larger number of observations for each angle of incidence such as the case of polarimetric data which measures both phase and amplitude. This technique could give accurate results up to a much higher level of noise. Similarly, in a simultaneity study where radars with different polarization will be looking at the earth from more than one platform to give only a few look angles, the present technique can be an effective tool for inverting the remotely sensed data.

7. REFERENCES

1. Lang, R, Electromagnetic Backscattering from Sparse Distribution of Lossy Dielectric Scatterers, Radio Sci, 16, 15-30, 1981.

2. Lang, R & S Sidhu, Electromagnetic Scattering from a Layer of Vegetation, IEEE Trans Geosci Remote Sens, GE-21, 62-71, 1983.

3. Karam, M and A Fung, Scattering from Randomly Oriented Discs with Application to Vegetation, Radio Sci, 18, 557-565, 1983.

4. Tsang, L, J Kong and R Shin, Radiative Transfer Theory for Active Remote Sensing of a Layer of Nonspherical Particles, Radio Sci, 19, 629-642, 1984.

5. Phillips, D, A Technique for Numerical Solution of Certain Integral Equations of the First Kind, JACM, 9, 84-97, 1962.

6. Twomey, S, On Numerical Solution of Fredholm Integral Equation of First Kind by Inversion of Linear System Produced by Quadrature, JACM, 10, 97-101, 1963.

7. Twomey, S, Introduction to the Mathematics of Inversion in Remote Sensing and Indirect Measurements, New York, Elsevier, 1977.

8. Lang, R, & H Saleh, Microwave Inversion of Leaf Area and Inclination Angle distribution from Backscattered Data, IEEE Trans GeoSci & Remote Sens, GE-23, 685-694, 1985.

9. Reagan, J, D Byren & B German, Bistatic Lidar: A Tool for Characterizing Atmospheric Particles: Part II, IEEE Trans GeoSci Remote Sens, GE-20, 236-243, 1982.

10. Mergenthaler, J, Inversion of Polarization Measurements at EUV Wavelengths to Recover Ozone and Aerosol Profile, Appl Opt, 23, 990-997, 1984.

11. Goel, N and R Thompson, Inversion of Vegetation Canopy Reflectance Models for Estimating Agronomic Variables. V. Estimation of Leaf Area Index and Average Leaf Angle Using Measured Canopy Reflectance, Remote Sensing of Envir, 16, 69-85, 1984.

12. Saleh, H and R Lang, Two Dimensional Inversion of Leaf Angle and Inclination Distributions from Backscattered Data, in Proc of the Symp on Multiple Scattering of Waves in Random Media and Random Rough Surfaces, 221-231, Technomic Pub, Lascaster, PA, 1987.

13. Lang, R, S Sekker & D LeVine, Scattering From a Random Layer of Leaves in Physical Optics Limit, Proc IGARSS Symp, Munich, TA-1, 3.1-3.6, 1982

14. Sekker, S, Resonant Scattering from Sparsely Distributed Lossy Dielectric Scatterers, Ph.D. Thesis, The George Washington University, Washington, DC, 1982.

15. The following typographical corrections should be made to [8]:(i)-replace $(n \cdot \alpha)\alpha$ by $(n \cdot \alpha)n$ in eq(4); (ii)- replace $2\pi A$ by 2π in eq(5); (iii)-delete $\int d\phi/2\pi$ from eq(27).

16. Morozov, V, Regularization of Incorrectly Posed Problems and the Choice of Regularization Parameter, USSR Comput Math and Math Phys, 6, 242-251, 1966(English Translation).

17. Miller, K, Least Square Method for Ill-posed Probems with Prescribed Bounds, SIAM J Math Anal, 1, 52-74, 1970.

18. Fung, A & F Ulaby, A Scatter Model for Leafy Vegetation, IEEE Trans Geosci Electron, GE-16, 281-286, 1978.

19. Smith, J, R Oliver & J Berry, A Comparison of Two Photographic Techniques for Estimating Foliage Angle Distribution, Aust J Bot, 25, 545-553, 1977.

20. Cimino, J, A Brandani, D Casey, J Rabassa & S Wall, Multiple Incidence Angle SIR-B Experiment Over Argentina IEEE Trans GeoSci & Remote Sens, GE-24, 498-509, 1986.

Table 1. Comparison of Different Cases and Their Errors (5% noise)

Polarization Type	ϵ_t	ϵ_q	ϵ_α
9-Pt. Horizontal	$.781 \times 10^{-2}$	$.200 \times 10^{-2}$	$.604 \times 10^{-2}$
9-Pt. Vertical	$.698 \times 10^{-2}$	$.200 \times 10^{-2}$	$.499 \times 10^{-2}$
9-Pt. Cross-Pol	$.804 \times 10^{-2}$	$.200 \times 10^{-2}$	$.610 \times 10^{-2}$
3-Pt. Horizontal	$.229 \times 10^{-1}$	$.186 \times 10^{-1}$	$.470 \times 10^{-2}$
3-Pt. Combined	$.233 \times 10^{-2}$	$.200 \times 10^{-2}$	$.617 \times 10^{-3}$

Figure Captions

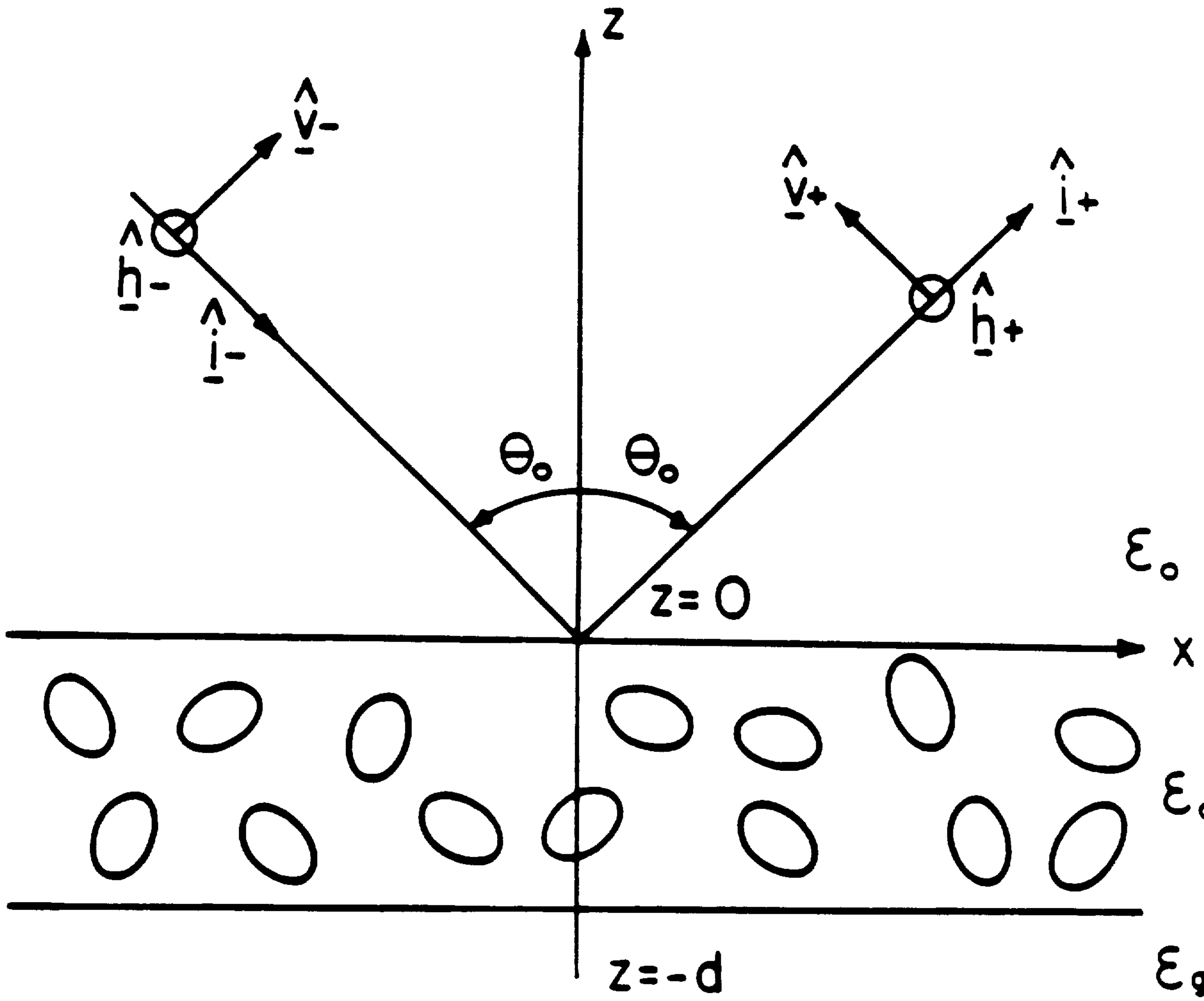
Figure 1. Soybean canopy model

Figure 2. Quadrature approximation of probability density

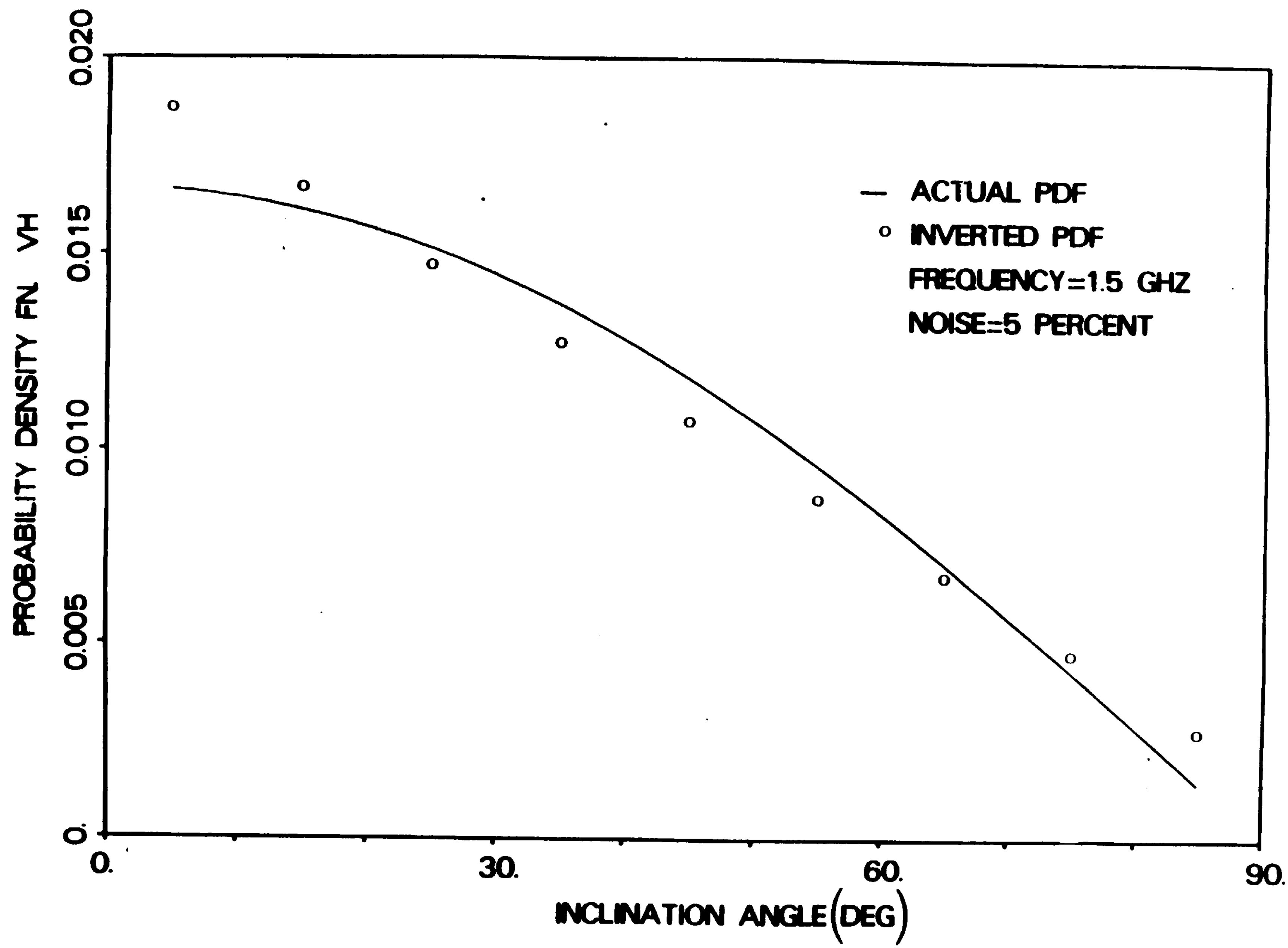
Figure 3. Backscattering coefficient vs. incident angle -9 point case (a) horizontal, (b) vertical, and (c) cross-polarization

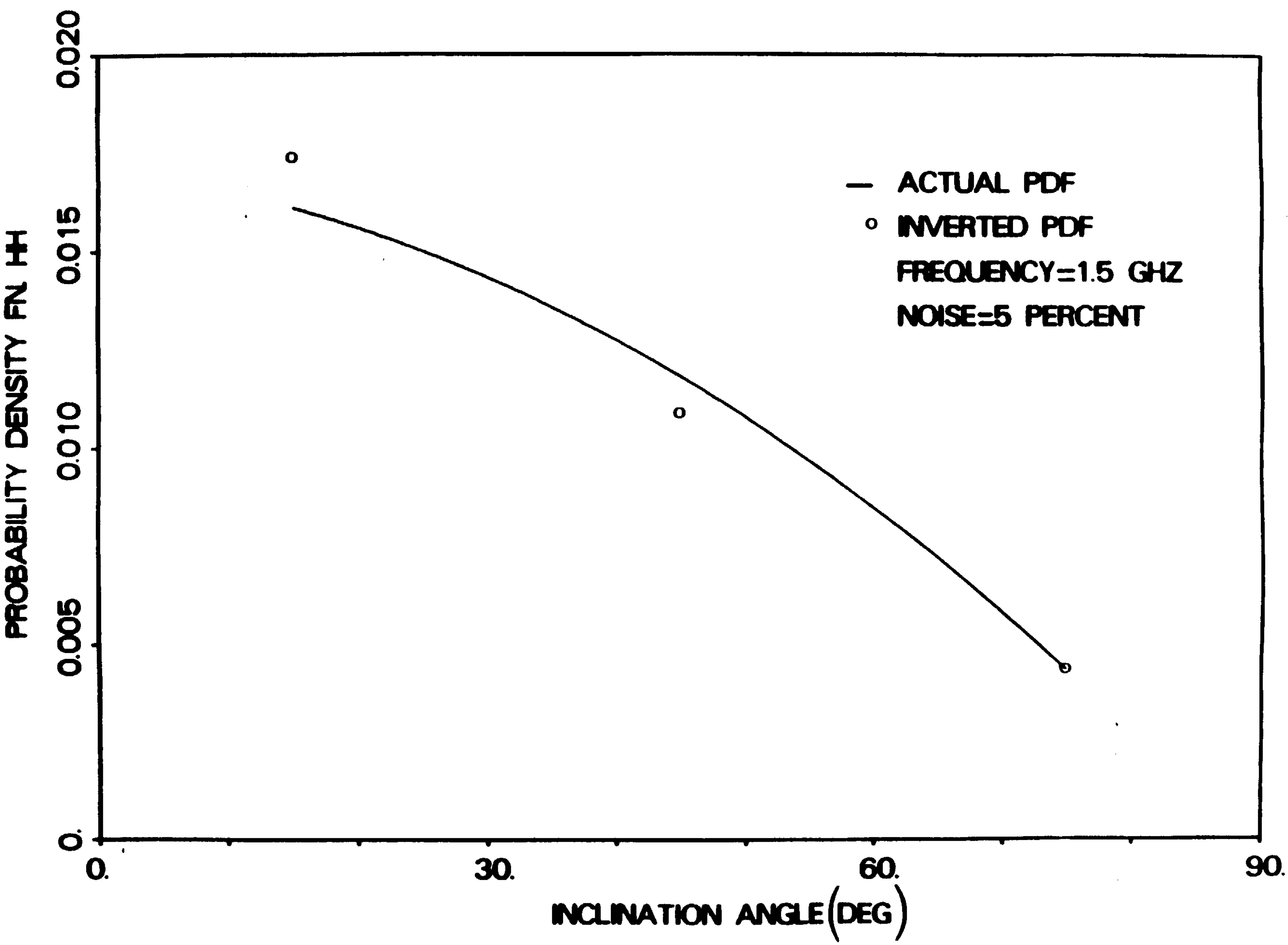
Figure 4. Probability density function vs. inclination angle -9 point case (a) horizontal, (b) vertical, and (c) cross-polarization

Figure 5. Probability density function vs. inclination angle -3 point case (a) single polarization (horizontal), (b) combined polarization

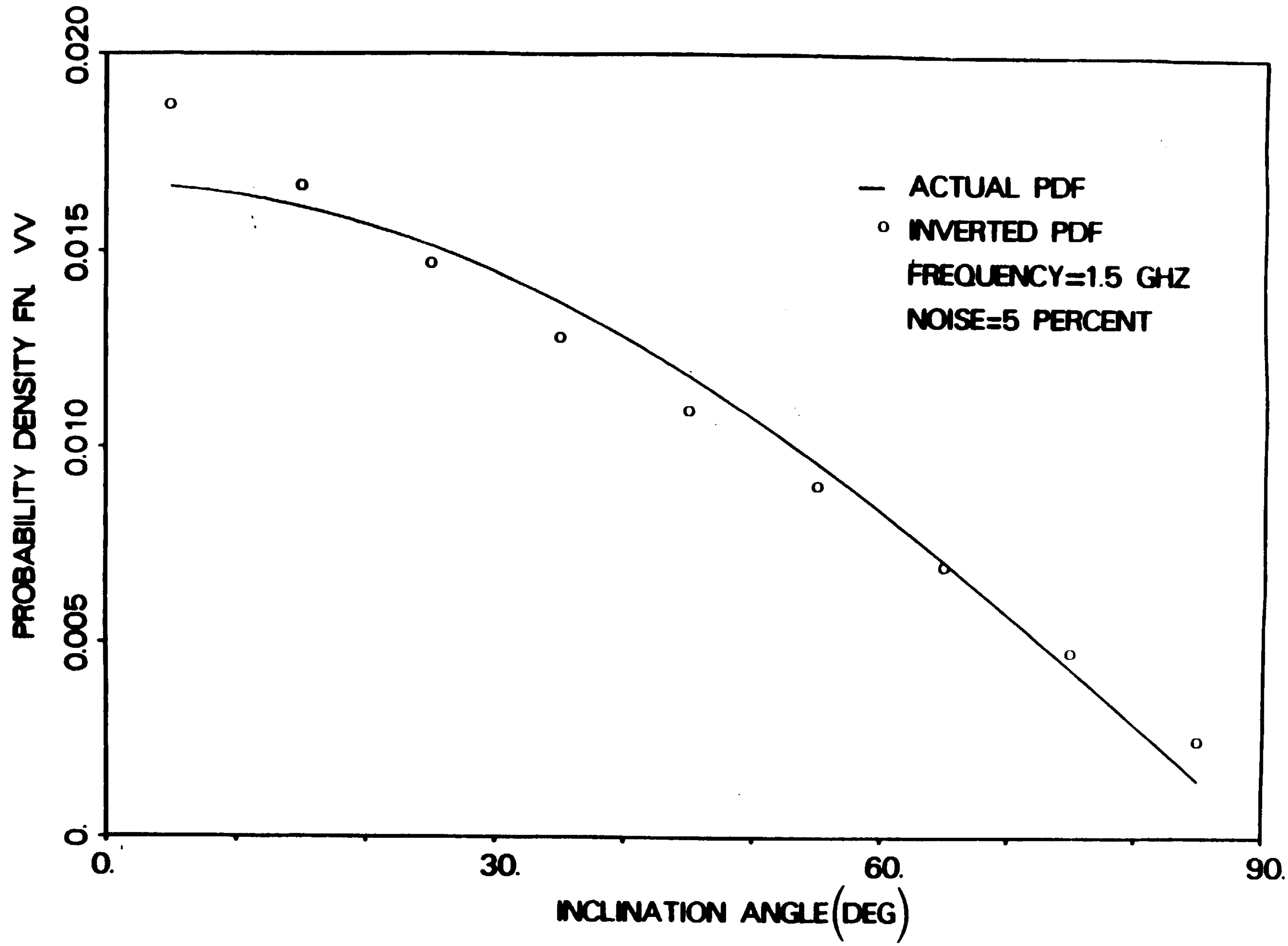


z
$\hat{\underline{v}}_-$
$\hat{\underline{h}}_-$
$\hat{\underline{i}}_-$
$\hat{\underline{v}}_+$
$\hat{\underline{i}}_+$
$\hat{\underline{h}}_+$
θ_o
θ_o
ε_o
z= 0
x
ε_o
z=-d
ε_g

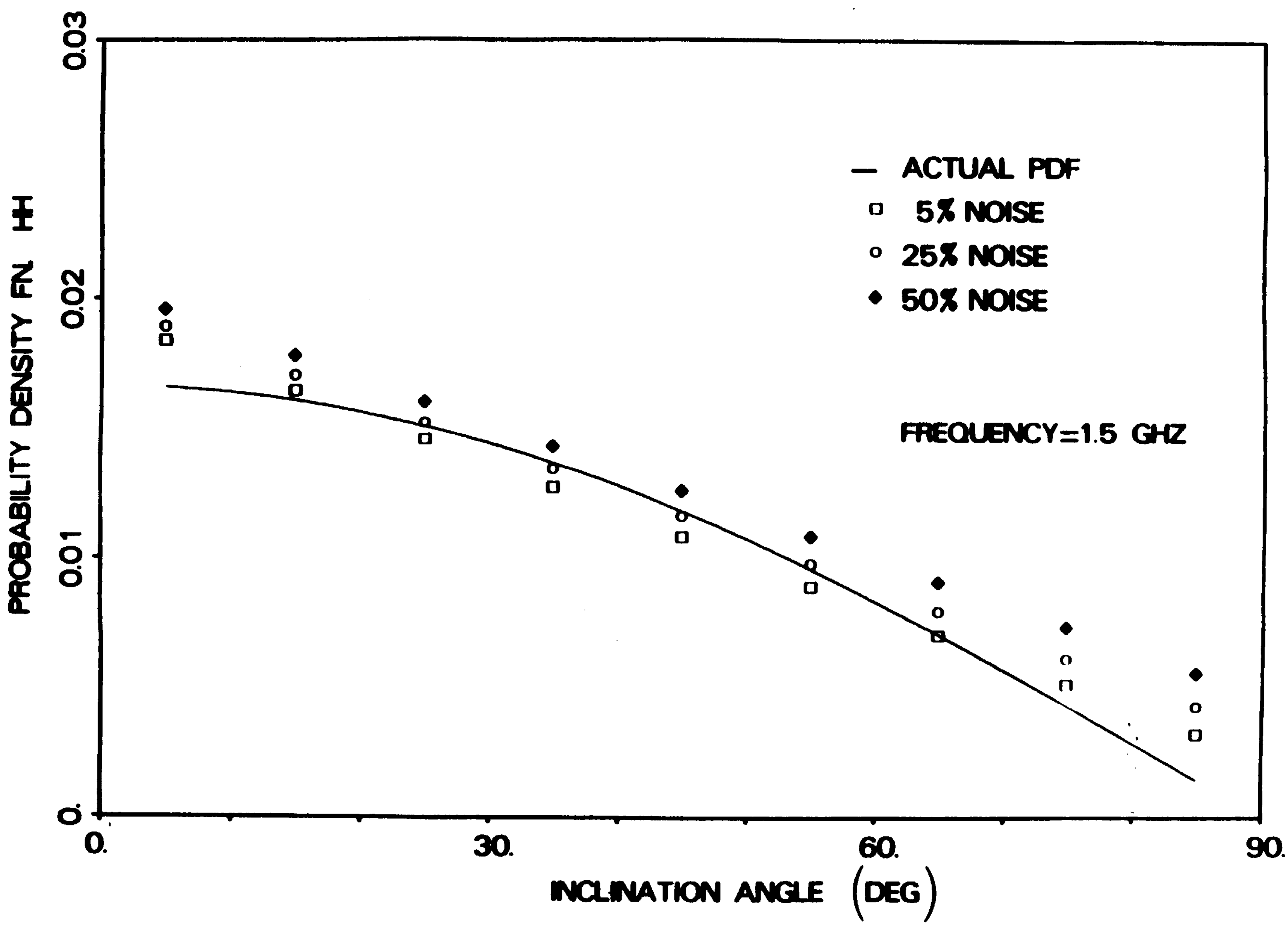

PROBABILITY DENSITY FN. VH
0.020
0.015
0.010
0.005
0.
0.
30.
60.
90.
INCLINATION ANGLE (DEG)
— ACTUAL PDF
o INVERTED PDF
FREQUENCY=1.5 GHZ
NOISE=5 PERCENT



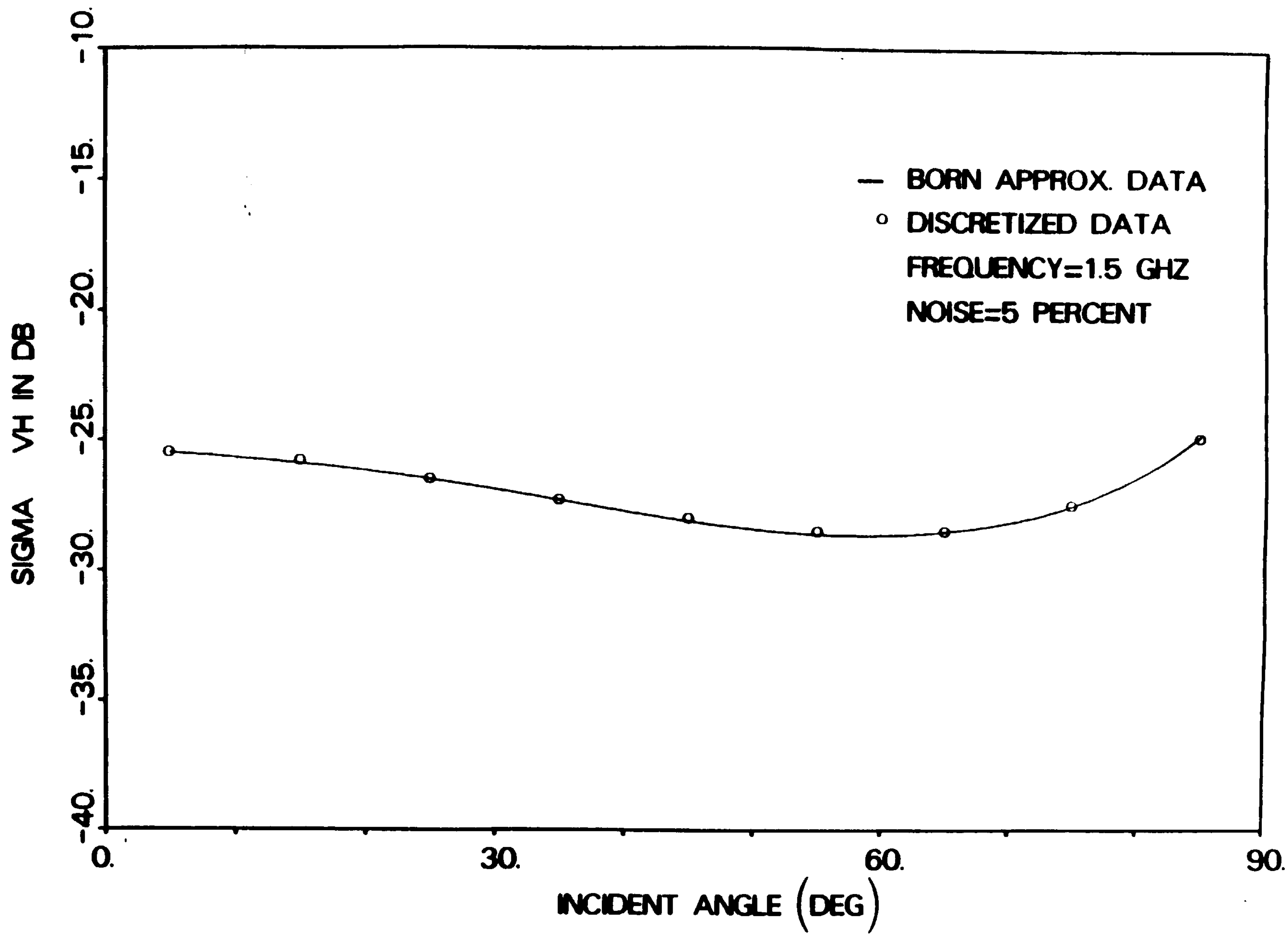
— ACTUAL PDF
o INVERTED PDF
FREQUENCY=1.5 GHZ
NOISE=5 PERCENT
0.020
0.015
0.010
0.005
0.
PROBABILITY DENSITY FN. HH
0.
30.
60.
90.
INCLINATION ANGLE (DEG)

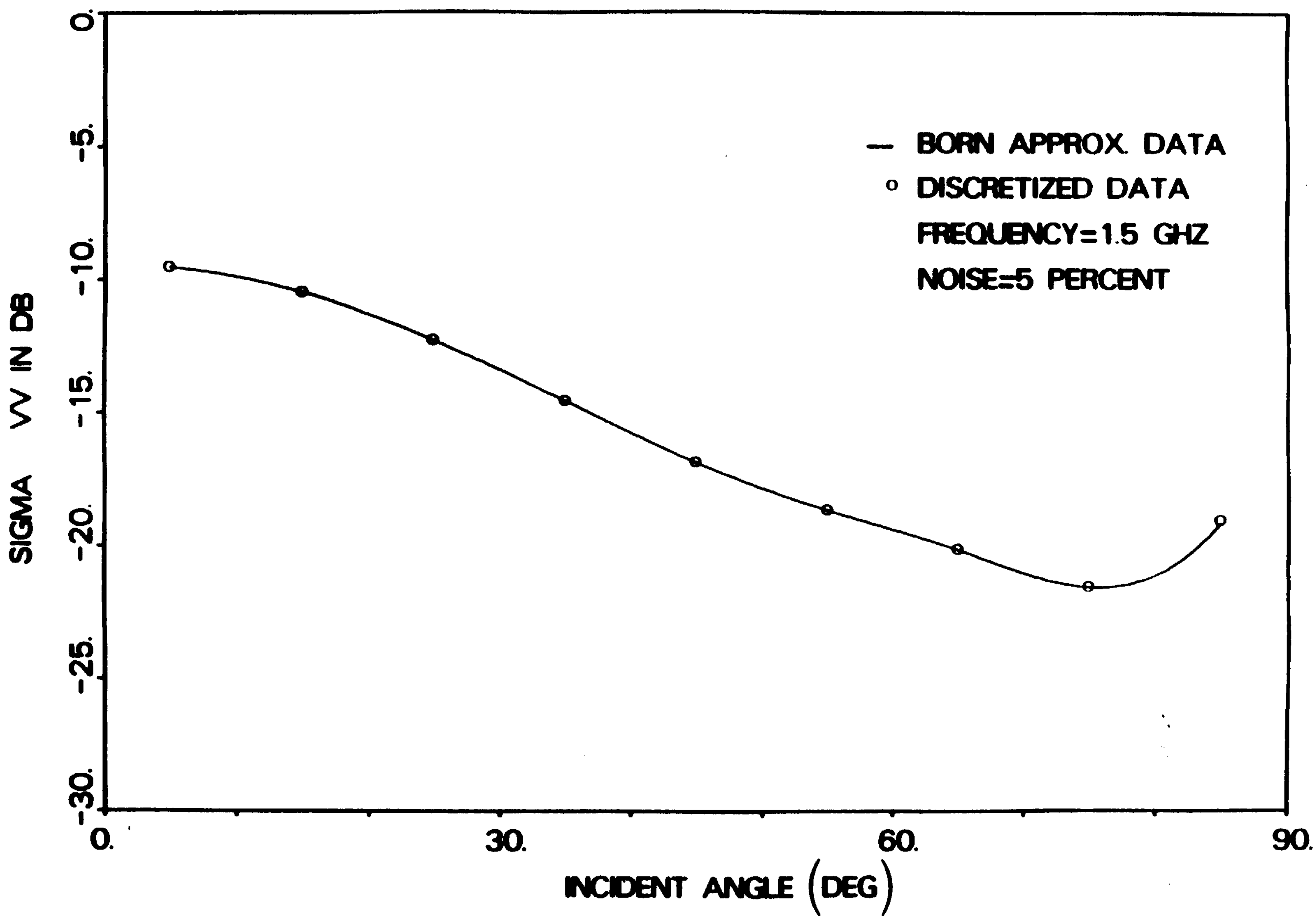


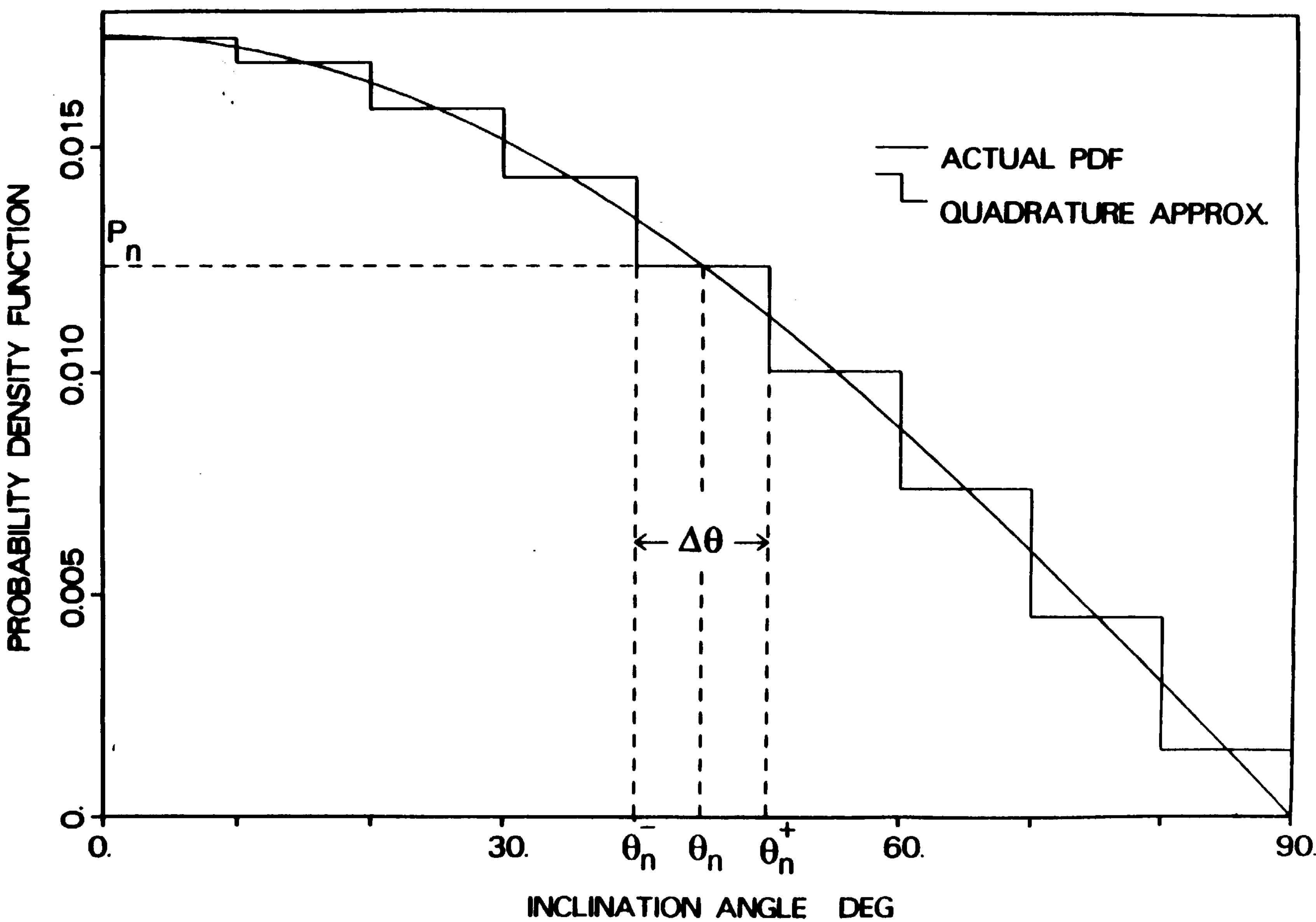
PROBABILITY DENSITY FN. VV
0.
0.005
0.010
0.015
0.020
0.
30.
60.
90.
INCLINATION ANGLE (DEG)
ACTUAL PDF
INVERTED PDF
FREQUENCY=1.5 GHZ
NOISE=5 PERCENT



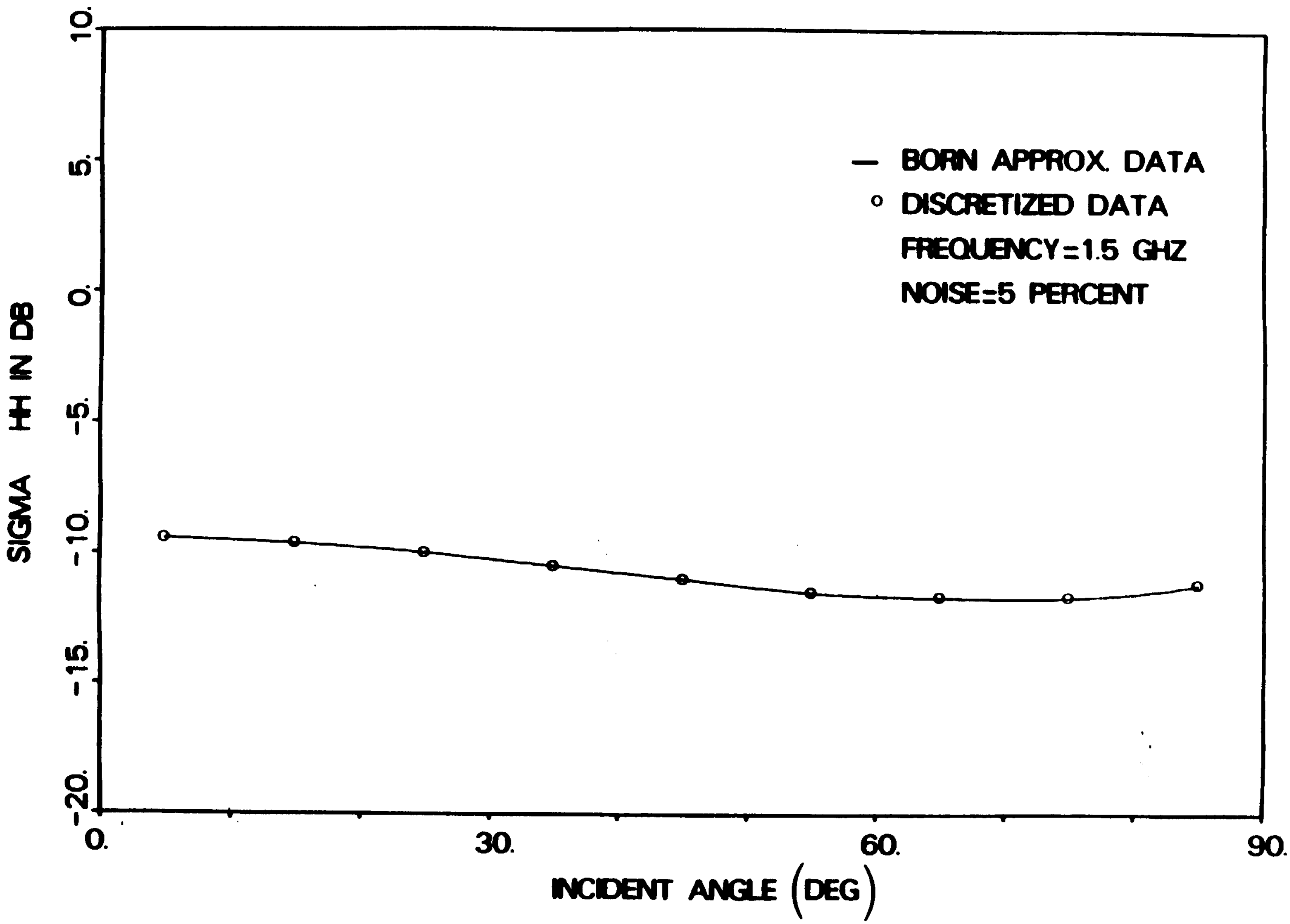
ACTUAL PDF
5% NOISE
25% NOISE
50% NOISE
FREQUENCY=1.5 GHZ
PROBABILITY DENSITY FN. HH
0.
0.01
0.02
0.03
0.
30.
60.
90.
INCLINATION ANGLE (DEG)

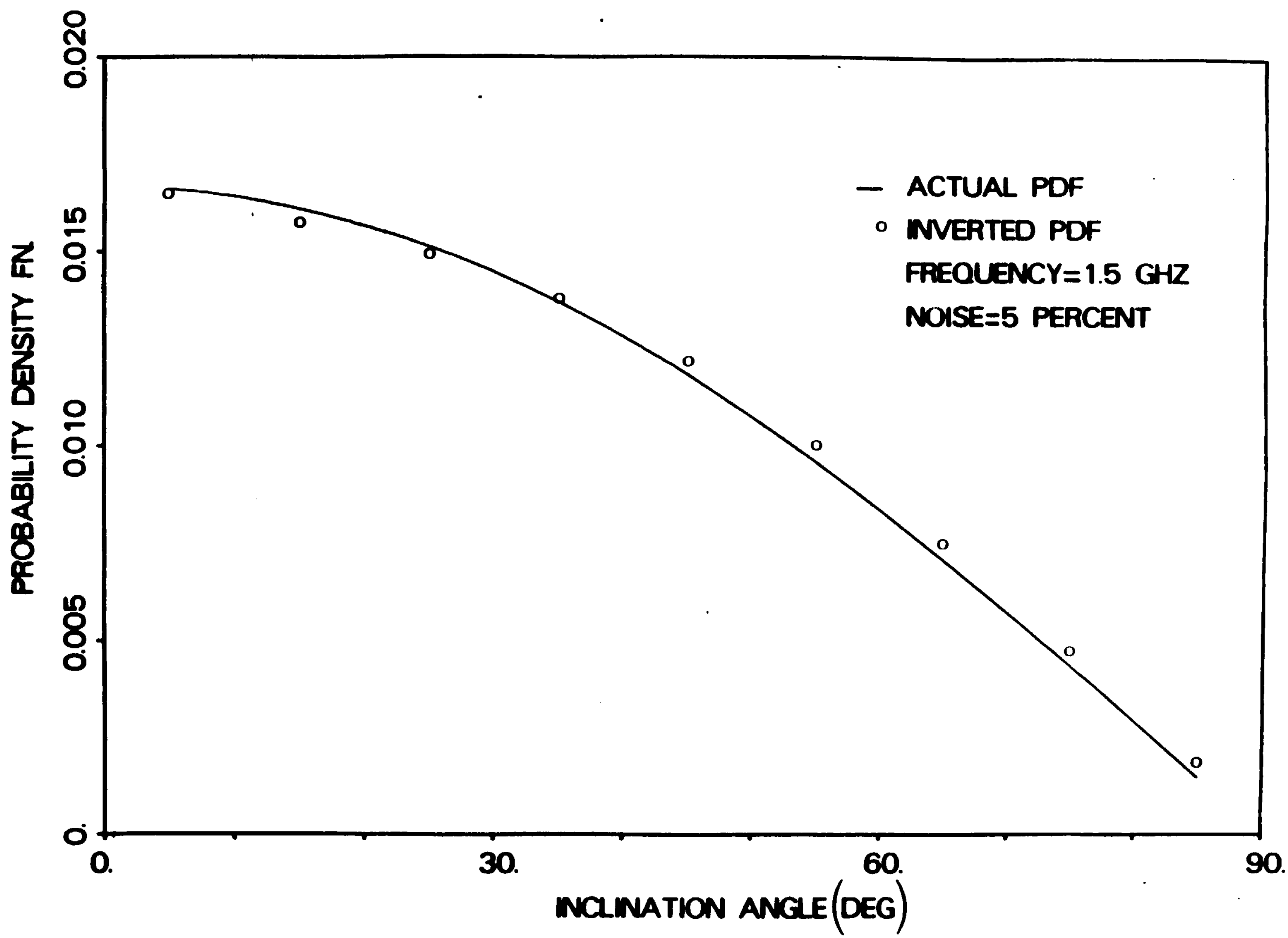

SIGMA VH IN DB
-10.
-15.
-20.
-25.
-30.
-35.
-40.
0.
30.
60.
90.
INCIDENT ANGLE (DEG)
— BORN APPROX. DATA
o DISCRETIZED DATA
FREQUENCY=1.5 GHZ
NOISE=5 PERCENT


— BORN APPROX. DATA
o DISCRETIZED DATA
FREQUENCY=1.5 GHZ
NOISE=5 PERCENT
SIGMA VV IN DB
0.
-5.
-10.
-15.
-20.
-25.
-30.
0.
30.
60.
90.
INCIDENT ANGLE (DEG)



PROBABILITY DENSITY FUNCTION
0.
0.005
0.010
0.015
P
n
ACTUAL PDF
QUADRATURE APPROX.
Δθ
0.
30.
θ
n
-
θ
n
θ
n
+
60.
90.
INCLINATION ANGLE DEG


BORN APPROX. DATA
DISCRETIZED DATA
FREQUENCY=1.5 GHZ
NOISE=5 PERCENT
SIGMA HH IN DB
INCIDENT ANGLE (DEG)
10.
5.
0.
-5.
-10.
-15.
-20.
0.
30.
60.
90.



PROBABILITY DENSITY FN.
0.
0.005
0.010
0.015
0.020
0.
30.
60.
90.
INCLINATION ANGLE (DEG)
ACTUAL PDF
INVERTED PDF
FREQUENCY=1.5 GHZ
NOISE=5 PERCENT